

Modelling Crop Insurance Based on Weather Index Using The Homotopy Analysis for American Put Option

Agus Sofian Eka Hidayat^{1*}, Monica Sandi Afa², Dedi Kurniawan³

¹Actuarial Department, President University, Indonesia, e-mail : agus.eka@president.ac.id

²Loyola Senior High School, Indonesia, e-mail : afa.monica.89@gmail.com

³Bussiness Administration Department, President University, Indonesia, e-mail : dedi.kurniawan@president.ac.id

ABSTRACT

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The crop insurance in Indonesia (AUTP) is much focused on the area impacted by flood, drought, and pest attack. The complication of the procedure to claim the loss must follow several conditions. The different approaches in the insurance sector, using weather index can be taken into consideration to produce a variety of insurance products. This insurance product used the American put option with the primary asset is the rainfall and the cumulative rainfall to exercise the claim, considering the optimal execution limit. The homotopic analysis is used to determine the valuation of the American put option, which also becomes the insurance premium. The case study is focused on areas experiencing a drought so that insurance claims can be exercise when the rainfall index value is below a predetermined limit. Considering the normality of the rainfall data, the calculation of insurance premium was done for the first growing season. The insurance premium is varies based on the optimal execution limit, while the calculation of profit is based on the optimum limit exercise and the minimum rainfall for the growing season, and its different depended on insurance claim acceptance limits.



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A. INTRODUCTION

The crop insurance in Indonesia (AUTP) is much focused on the area impacted by flood, drought, and pest attack. The status quo right now, that to join AUTP the farmers should pay the premium Rp 180000 per hectare which is subsidized for about 144000 by the government, where the farmers will receive Rp 600000 per Hectare if the farmers having crop failure minimum 75%, affected by flood, pest, and drought. it has premium insurance complication of the procedure to claim the loss must follow several conditions. According to Hidayat & Gunardi (2019) there are some tendency of the farmers is not interested to join the AUTP because the complexity of the claim process as well as the premium is still not reasonable. Thus, it's important to adjust new types of insurance, which can give another option to farmer or group of farmers for insuring their lands.

Insurance based on rainfall is becoming new trend in development country such as in China, Australia, and Iran (Adeyinka et al., 2016). Rainfall can be used for calculating the insurance premium because its dependency with the yield (Hidayat & Gunardi, 2019). There are several techniques that can be used to determine the premium of insurance premium of crop insurance based of cumulative rainfall such as copula (Bokusheva, 2018), weather index (Adeyinka et al., 2016) and American put option using Black Scholes Model (Putri et al., 2017). As we can see that option concept is common to be implemented for determining the premium of the insurance.

American put option is an option that gives the right not obligation for the option holder to sell the asset at certain prices at any time, it is different with European put option which must be exercised at maturity time. The problems in the valuation of American put option price is related to the free limitation problems. To overcome the problems analytically, Cheng et al., (2010), Dyke & Liao (2012) used the homotopy analysis method to find the solution of pricing the American put option and it is also can be implemented for European option (Fadugba, 2020). Homotopy analysis method can be used to solve non-linear problems and it gives the freedom to choose the type of equation from linear problems and basic function from the solution. The implementation of homotopy method is common in sector such as science, finance, and engineering. The homotopy analysis is also can be used to do pricing under option using Levy process (Sakuma & Yamada, 2014) and stochastic volatility (Park & Kim, 2011).

In this research we try to explore more about the implementation of homotopy analysis method which also have been introduced by Zhu (2006) to analyse the solution of the price of American put option. We try to assess the price of American put option for calculating crop insurance premium which used the cumulative rainfall as the basic assets. The result is expected able to give another spectrum of implementation crop insurance in Indonesia, hence there is various type of crop insurance that can be choose by the farmers in the future to insure their assets.

B. LITERATURE REVIEW

1. American put Option

American put option gives the right not obligation to the option holder to sell the asset at certain time from the beginning up to the maturity time (Higham, 2004). The calculation of American option price using the Black-Scholes model involving the log normal return of the assets.

$$R_t = \ln \left(\frac{S_t}{S_{t-1}} \right) \tag{1}$$

where R_t is return assets at time t , S_t is stock price at time t , and S_{t-1} is the stock price at time $t - 1$. For the implementation for this research, the term of log normal return of stock is modified with the log normal ratio of cumulative rainfall.

According to Hull (2009) the volatility of stock, σ , is the uncertainty from the return of the stock. There are two types of volatility which are historical volatility and implied volatility. Historical volatility of the stock is estimated from the historical data of the stock prices.

2. Black Scholes formula

Assume that the asset prices follow the formula of asset price movement model,

$$S_t = S_0 \exp \left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma \sqrt{t} Z \tag{2}$$

where S_t is the price of assets at time t , S_0 is the asset price at time 0, μ is the mean of asset return, σ is the volatility of the asset price, both μ and σ constant, and $Z \sim N(0,1)$. The Black-Scholes option pricing formula for European type put option with strike price X , maturity time T , and risk-free interest rate r is,

$$V_E = X \exp(-rT) N(-d_2) - S_0 N(-d_1) \tag{3}$$

given $d_1 = \frac{\ln(\frac{S_0}{X}) + (r + \frac{1}{2} \sigma^2) T}{\sigma \sqrt{T}}$, $d_2 = d_1 - \sigma \sqrt{T}$, and $N(x)$ is cumulative distribution function from standard normal.

3. Decomposition of American Put Option

In American put option, if P_t denoted the price of the option at time $t \in [0, T]$, for each time $t \in [0, T]$ there exist an optimal execution limit, B_t , which is optimal to exercise the option when S is figure 1 or above B_t .

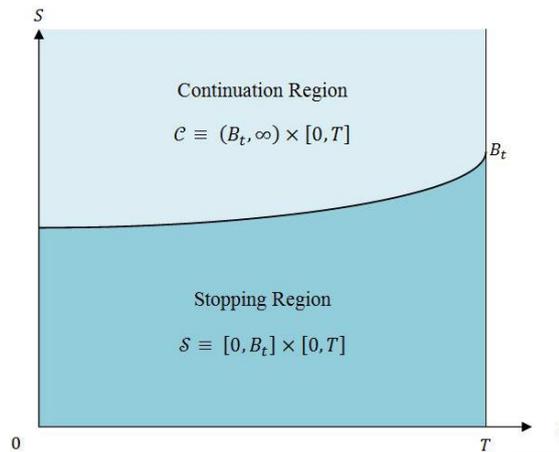


Figure 1. Domain Function of American Put Option

Graduate & Myneni (1992) explained that in continues area \mathcal{C} , the price of American put option, P_0 , can be decomposed from European put option, p_0 , and the premium for early exercises, e_0 :

$$P_0 = p_0 + e_0 \tag{4}$$

where $e_0 = rX \int_0^T \exp(-rt) N\left(\frac{\ln\left(\frac{B_t}{S_0}\right) - e_2 t}{\sigma\sqrt{t}}\right) dt$ and $e_2 = r - \frac{\sigma^2}{2}$.

4. Homotopy Analysis Method

Homotopy between two continues function $f(x)$ and $g(x)$ from topology space X to topology space Y is defined as continues function $\mathcal{H}: X \times [0, 1] \rightarrow Y$ from multiplication of space X with interval $[0,1]$ to Y such that if $x \in X$ then $\mathcal{H}(x; 0) = f(x)$ and $\mathcal{H}(x; 1) = g(x)$. In topology $f(x)$ and $g(x)$ is called homotopic, $\mathcal{H}: f(x) \sim g(x)$. From the set of real function $C[a, b]$ if $f \in C[a, b]$ continually deformed into $g \in C[a, b]$ then we can perform homotopy $\mathcal{H}: f(x) \sim g(x)$ with $\mathcal{H}(x; q) = (1 - q)f(x) + qg(x), x \in [a, b]$. The parameter $q \in [0,1]$ from homotopy is called homotopy parameter (Dyke & Liao, 2012). Furthermore, for each homotopy,

$$\mathcal{H}(x; q) = (1 - q)f(x) + qg(x), x \in [a, b]$$

we can define the first derivative of homotopy,

$$\frac{\partial \mathcal{H}(x; q)}{\partial q} = g(x) - f(x), q \in [0, 1] \tag{5}$$

Which describe the ratio or rate continue deformation from $f(x)$ to $g(x)$.

Dyke & Liao (2012) explained that the nonlinear function ε_1 which has at least one solution $u(z, t)$ with z and t are independent spatial and temporal variable, respectively. Given, $q \in [0,1]$ denote the homotopy parameter and $\bar{\varepsilon}(q)$ is the null deformation equation which is connected the original ε_1 and ε_0 with approximation $u_0(z, t)$. Assume that the null deformation of $\bar{\varepsilon}(q)$ performed well which make the solution $\phi(z, t; q)$ exists, for $q = 0$. Hence the null deformation equation is $\phi(z, t; 0) = u_0(z, t)$ while for $q = 1$, $\bar{\varepsilon}(q)$ equivalent with ε_1 thus $\phi(z, t; 1) = u_0(z, t)$.

The Maclaurin sequence from $\phi(z, t; q)$ is

$$\phi(z, t; q) \sim u_0(z, t) + \sum_{n=1}^{+\infty} u_n(z, t) q^n \tag{6}$$

where

$$u_n(z, t) = \frac{1}{n!} \left. \frac{\partial^n \phi(z, t; q)}{\partial q^n} \right|_{q=0} = \mathcal{D}_n[\phi(z, t; q)]. \tag{7}$$

C. RESEARCH METHODS

Let $V(S, t)$ be the price of the American put option, where X is the strike price of the option, S is the price of the underlying asset, t is the time, r is the risk-free interest rate and σ is the volatility. In the American option, there is a critical price for the asset, $B(t)$, which makes optimum to exercise the American put option when S in or below the $B(t)$. For $S < B(t)$, the price of the option is $V(S, t) = X - S$, while for $S > B(t)$, $V(S, t)$ will satisfy the Black-Scholes,

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0 \tag{8}$$

which is depend on the smooth pasting condition at exercise limit $B(t)$

$$\begin{aligned} \lim_{S \rightarrow B(t)} V(S, t) &= X - B(t), \\ \lim_{S \rightarrow B(t)} \frac{\partial V}{\partial S} &= -1 \end{aligned} \tag{9}$$

With the upper boundary condition $\lim_{S \rightarrow \infty} V(S, t) = 0$ and terminal condition $\lim_{t \rightarrow T} V(S, t) = \max\{X - S, 0\}$.

Define $\tau \equiv T - t$ as the length of time for maturity drive the formula of option price as

$$V(S, \tau) = V_E(S, \tau) + X \int_0^\tau r \exp(-r\xi) N(-d_{\xi,2}) d\xi \tag{10}$$

where $V_E(S, \tau) = X \exp(-r\tau) N(-d_2) - SN(-d_1)$ is the price of European put option which is associated with

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)(\tau)}{\sigma\sqrt{\tau}}, \\ d_2 &= d_1 - \sigma\sqrt{\tau} \\ d_{\xi,1} &= \frac{\ln\left(\frac{S}{B(\tau-\xi)}\right) + \left(r + \frac{1}{2}\sigma^2\right)(\xi)}{\sigma\sqrt{\xi}}, \\ d_{\xi,2} &= d_{\xi,1} - \sigma\sqrt{\tau} \end{aligned} \tag{11}$$

$N(x)$ is cumulative distribution function and $\tau \equiv T - t$ is variable of time until maturity. From (10) the option price $V(S, \tau)$ can be obtained if $B(\tau)$ is given. Thus, the value of $B(\tau)$ is essential for this problem (Graduate & Myneni, 1992).

Equation (8) and (9) construct the form a differential system where the solutions give the price of American option at any underlying asset, S , and any time, t , before maturity (T). When $\sigma \neq 0$, defined the dimensionless variable.

$$V^* = \frac{V}{X}; S^* = \frac{S}{X}; \tau^* = \tau \frac{\sigma^2}{2}; \gamma = \frac{2r}{\sigma^2} \tag{12}$$

By eliminating the *, we can obtain the dimensionless equation,

$$-\frac{\partial V}{\partial \tau} + S^2 \frac{\partial^2 V}{\partial S^2} + \gamma S \frac{\partial V}{\partial S} - \gamma V = 0. \tag{13}$$

Under Homotopy analysis, we create two continue variance $\phi(S, \tau; q)$ and $\Lambda(\tau; q)$, for q from 0 to 1, $\phi(S, \tau; q)$ varies continuously from the initial guess $V_0(S, \tau)$ to the solution $V(S, \tau)$ while $\Lambda(\tau; q)$ varies continuously from the initial guess $B_0(\tau)$ to the optimal execution $B(\tau)$. That variance is constructed from the null deformation equation:

$$-\frac{\partial \phi(S, \tau; q)}{\partial \tau} + S^2 \frac{\partial^2 \phi(S, \tau; q)}{\partial S^2} + \gamma S \frac{\partial \phi(S, \tau; q)}{\partial S} - \gamma \phi(S, \tau; q) = 0 \tag{14}$$

with domain $\Lambda(\tau; q) \leq S < \infty, 0 \leq \tau \leq \tau_{exp}$. For $q = 1$, we have $\phi(S, \tau; 1) = V(S, \tau)$ and $\Lambda(\tau; 1) = B(\tau)$. When $q = 0$, $\Lambda(\tau; 0) = B_0(\tau)$, from (14) we have,

$$-\frac{\partial V_0(S, \tau)}{\partial \tau} + S^2 \frac{\partial^2 V_0(S, \tau)}{\partial S^2} + \gamma S \frac{\partial V_0(S, \tau)}{\partial S} - \gamma V_0(S, \tau) = 0 \tag{15}$$

With prior condition

$$\begin{aligned} V_0(S, \tau) &= 0, \\ \frac{\partial V_0(S, \tau)}{\partial S} &= -1, \text{ for } S = B_0(\tau), \\ \lim_{S \rightarrow \infty} V_0(S, \tau) &= 0. \end{aligned} \tag{16}$$

For simplification take $B_0(\tau) = 1$ as an initial guess of the optimal execution limit. The null deformation creates two variety or deformation which are homotopy,

$$\phi(S, \tau; q): V_0(S, \tau) \sim V(S, \tau), \quad \Lambda(\tau; q): B_0(\tau) \sim B(\tau). \tag{17}$$

Further by expanding the $\phi(S, \tau; q)$ and $\Lambda(\tau; q)$ using the the Maclaurin sequences for $q \in [0,1]$ we construct the Maclaurin-homotopy,

$$\phi(S, \tau; q) = V_0(S, \tau) + \sum_{n=1}^{\infty} V_n(S, \tau) q^n \tag{18}$$

and

$$\Lambda(\tau; q) = B_0(\tau) + \sum_{n=1}^{\infty} B_n(\tau) q^n \tag{19}$$

with

$$\begin{aligned} V_n(S, \tau) &= \left. \frac{1}{n!} \frac{\partial^n \phi(S, \tau; q)}{\partial q^n} \right|_{q=0} = D_n[\phi(S, \tau; q)], \\ B_n(\tau) &= \left. \frac{1}{n!} \frac{\partial^n \Lambda(\tau; q)}{\partial q^n} \right|_{q=0} = D_n[\Lambda(\tau; q)], \end{aligned} \tag{20}$$

The null deformation equation contains convergency control parameter c_0 . Assume that c_0 selected carefully thus the sequence is absolute convergence in $q = 1$. Hence the solution of the homotopy sequence is

$$V(S, \tau) = V_0(S, \tau) + \sum_{n=1}^{\infty} V_n(S, \tau) \tag{21}$$

$$B(\tau) = B_0(\tau) + \sum_{n=1}^{\infty} B_n(\tau). \tag{22}$$

By substituting the sequence (18) and (19) to (15 and (16) and equalizing the like-power from q , we obtain the n- deformation ($n \geq 1$),

$$-\frac{\partial V_n(S, \tau)}{\partial \tau} + S^2 \frac{\partial^2 V_n(S, \tau)}{\partial S^2} + \gamma S \frac{\partial V_n(S, \tau)}{\partial S} - \gamma V_n(S, \tau) = 0 \tag{23}$$

which depends on initial condition $V_n(S, 0) = 0$ and $\lim_{S \rightarrow \infty} V_n(S, \tau) = 0$. If ' denote the differential respect to S , by expanding $(S, \tau; q)$, $\phi'(S, \tau; q)$ to the Maclaurin sequence under q

$$\phi(S, \tau; q) = V_0(B_0, \tau) + \sum_{n=1}^{\infty} [V_n(B_0, \tau) + f_n(\tau)] q^n \tag{24}$$

and

$$\frac{\partial \phi(S, \tau; q)}{\partial S} = V'_0(B_0, \tau) + \sum_{n=1}^{\infty} [V'_n(B_0, \tau) + g_n(\tau)] q^n, \tag{25}$$

with

$$f_n(\tau) = \sum_{j=1}^{n-1} \alpha_{j,n-j}(\tau), \quad g_n(\tau) = \sum_{j=1}^{n-1} \beta_{j,n-j}(\tau) \tag{26}$$

where it follows the following explicit definition,

$$\begin{aligned} \alpha_{n,i}(\tau) &= \sum_{m=1}^i \psi_{n,m}(\tau) \mu_{m,i}(\tau); \\ \beta_{n,i}(\tau) &= \sum_{m=1}^i (m+1) \psi_{n,m+1}(\tau) \mu_{m,i}(\tau), \quad i \geq 1; \\ \psi_{n,0}(\tau) &= V_n(B_0, \tau); \end{aligned} \tag{27}$$

$$\psi_{n,m}(\tau) = \frac{1}{m!} \left. \frac{\partial^m V_n(S, \tau)}{\partial S^m} \right|_{S=1}$$

and recursion formula

$$\begin{aligned} \mu_{1,n}(\tau) &= B_n(\tau) \\ \mu_{m+1,n}(\tau) &= \sum_{i=m}^{n-1} \mu_{m,i}(\tau) B_{n-i}(\tau). \end{aligned} \tag{28}$$

Furthermore, we obtain $\frac{\partial V_n(S, \tau)}{\partial S} = -g_n(\tau)$ when $S = B_0(\tau) = 1$ for $n \geq 1$. Using the similar process, we also obtain

$$B_n(\tau) = \begin{cases} c_0 V_0(B_0, \tau), & n = 1 \\ B_{n-1}(\tau) + c_0 [B_{n-1}(\tau) + V_{n-1}(B_0, \tau) + f_{n-1}(\tau)], & n > 1 \end{cases} \tag{29}$$

with $B_0 = 1$. To simplify the equation, define $g_0(\tau) = 1$. Using the Laplace transformation from * we obtain,

$$\mathcal{L}_T \left[\frac{\partial V_n(S, \tau)}{\partial \tau} \right] = \zeta \mathcal{L}_T [V_n(S, \tau) - V_n(S, 0)] = \zeta \widehat{V}_n(S, \zeta) \tag{30}$$

and

$$\mathcal{L}_T \left[\frac{\partial^m V_n(S, \tau)}{\partial S^m} \right] = \frac{\partial^m \widehat{V}_n(S, \zeta)}{\partial S^m} \tag{31}$$

for $m \geq 0$. By Implementing the Laplace theorem, the differential equation become,

$$S^2 \frac{\partial^2 \widehat{V}_n}{\partial S^2} + \gamma S \frac{\partial \widehat{V}_n}{\partial S} - (\gamma + \zeta) \widehat{V}_n = 0 \tag{32}$$

and it depends on $\frac{\partial \widehat{V}_n}{\partial S} = -\widehat{g}_n(\zeta)$ for $S = 1$ and $\lim_{S \rightarrow \infty} \widehat{V}_n(S, \zeta) = 0$. Hence the differential equation has closed form solution,

$$\widehat{V}_n(S, \zeta) = \widehat{K}(S, \zeta) \widehat{g}_n(\zeta) \tag{33}$$

with $\widehat{K}(S, \zeta) = -\frac{S^2}{\lambda}$, $\lambda = \frac{1 - \gamma - \sqrt{4\zeta(1+\gamma)^2}}{2}$.

Consider $\widehat{g}_n(\zeta) = \mathcal{L}_T [\widehat{g}_n(\zeta)]$ as the Laplace transformation from $g_n(\zeta)$, by using the inverse of Laplace transformation, we obtain,

$$V_n(S, \tau) = \mathcal{L}_T^{-1} [\widehat{K}(S, \zeta) \widehat{g}_n(\zeta)] \tag{34}$$

In the process of the approximation of N-homotopy from $B(\tau)$ dimensionless in polynomial $\sqrt{\tau}$ until $o(\tau^M)$ explicitly by

$$B(\tau) \approx \sum_{m=0}^N B_m(\tau) = \sum_{k=0}^{2M} b_k (\sqrt{\tau})^k \tag{35}$$

With coefficient of b_k depends on $\gamma = 2r/\sigma^2$ and convergency control parameter c_0 . In the process of modification, the approximation of $B(\tau)$, we construct $z = \sqrt{\tau}$, further by using Pade approximation for $\sum_{n=0}^{2M} b_n z^n$ centered at $z = 0$ with degree $[M, M]$. Further, by changing the value of z with $\sqrt{\tau}$ we do have approximation from $B(\tau)$ until $o(\tau^M)$

D. RESULTS AND DISCUSSION

Data that used in this research is 10 days (*dasarian*) cumulative rainfall for Panggang, Yogyakarta. Panggang is chosen as the research focus because the agriculture sector is much depending on rainfall (Badan Pusat Statistik, 2015). The rainfall is calculated using millimeter measurement (mm) from 1998 to 2012. The commodity paddy is the focus for the insurance premium calculation which normally takes 120 days or 12 *dasarian* from the beginning up to harvesting. According to the integrated planting calendar, two planting times are used Planting Season I (MT I) and Planting Season II (MT II). The planting season I start from October III to February II, and while the planting season II start from February I to May II.

The assumption of normality data is used for the ln ration of the cumulative rainfall, because in Black-Scholes model we need to have normal data on the calculation. We tried to exercise through normality

statistics using Kolmogorov Smirnov test and the results for α the MT I and MT II has $p - value$ 0.526 and 0.001 respectively. Thus, ratio of the cumulative rainfall is normally distributed in MT I.

The premium is calculated for three acceptance limit value of claim which are 10th Quantile, 25th Quantile and 50th Quantile. The risk-free rate using the Indonesian Bank rate (SBI) and the volatility adjusted from the historical data of the cumulative rainfall. Based on the calculation the premium insurance from the crop insurance shown in the table 1.

Table1. Acceptence limit

Description	Data
District	Panggung MT I
10 th Quantile	64.77 mm
25 th Quantile	86.61 mm
50 th Quantile	112.86 mm
Interest Rate (r)	0.065
Volatility (σ)	0.69
Insurance period	120 days
Beginning Rainfall (S_0)	149 mm
Amount of Claim per mm	RP 100000

From the data above the calculation of premium/price using the American option using analysis of homotopy method, for each optimal execution with probability of having profit is shown in the table 2.

Table2. Premium and profit Calculation

Description	Data		
Limit acceptance claim (mm)	10 th - 64.77	25 th - 86.61	50 th - 112.86
Limit optimum execution (mm)	21.35	28.54	37.194
Insurance Premium	195,800	613,400	1,500,300
Profit	432,300	5,806,400	7,566,400
Probability	0.08	0.28	0.56
Ratio of Premium/Profit	0.453	0.106	0.2

E. CONCLUSSION AND SUGGESTION

Homotopy analysis method can be implemented to overcome problems in American option put, to determine optimal execution limit. The American put option can be develop not only for pricing the assets but also for pricing the crop insurance by using the cumulative rainfall.

There is possibility for the implementation on cumulative rainfall for crop insurance in Indonesia. The used of cumulative rainfall still adjustable comparing with the insurance premium and the amount of profit that will be taken. The number of profits is increase according to the premium. The earlier execution the probability of the claim that happened is smaller compare to the other execution limit. However, its still questionable weather the farmer groups are willing to join the insurance, as the AUTP is also undergoing and given subsidized by the government. Hence, it is necessary to conduct more study survey to portray what type of insurance that is needed by the farmers.

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