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Mathematical Modelling and Simulation Strategies for Controlling Damage to Forest Resources Due to Illegal Logging

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ABSTRACT

Forests are one of the natural resources that provide many benefits for the welfare of living things. The dense population causes people to depend more on forest resources. One of them is illegal logging. Various strategies to control forest damage due to illegal logging have been carried out, namely by direct handling to improve damaged conditions while preventing the recurrence of forest damage. The purpose of this research is to build a mathematical model of forest resource damage control strategies due to illegal logging, determine assumptions, formulate the model, and conduct analysis and problem solving including: determining the equilibrium point, determining the stability analysis of the equilibrium point, and conducting numerical simulations of the equilibrium point. The last step is to interpret the results of the analysis obtained and make conclusions. Based on the research and simulation results of the model, it can be concluded that taking into account the variable of forest resource damage control strategy due to illegal logging, the result shows that if the density of forest resources has been affected by the disturbance of population density around the forest, it is necessary to have a forest resource damage control strategy in order to compensate for the people around the forest who do a lot of illegal logging. In order to maintain the forest so that the forest does not quickly become extinct and can overcome drought, prevent flooding, maintain groundwater quality, protect animals, reduce air pollution, climate control, reduce dust particles, prevent the greenhouse effect, supply natural fertilizers, prevent erosion, and maintain springs.

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A. INTRODUCTION

Forests are one of the natural resources that provide many benefits for the welfare of living things. Forest resources consist of trees, grass, animals, water, fruits, and medicinal plants. Forests, apart from being a source of wood production, also have a role as an oxygen producer, provider of water sources, a place for plants and animals to live, and prevent the onset of global warming, so they must be properly maintained and preserved (Zainuddin & Tahnur, 2018).

In 2020, according to The Statistics Office of North Central Timor District (2021), the population of North Central Timor District was 259,829 people, with an area of $2,669 km^2$ and a population density of 95 people/km^2. The dense population causes the need for land for housing development, infrastructure, agriculture is also high so that the community relies more on forest resources. The attitude of the community in utilizing the forest is different, influenced by the interest s of each individual in meeting their life needs so that the community utilizes forest resources excessively.

Over-utilizing the forest has a negative impact on the environment and the welfare of the community. One of the factors of forest destruction is illegal logging, which is not only carried out by communities around the forest, but also by entrepreneurs who utilize the forest improperly. Increasingly severe forest damage causes many other natural disasters such as floods, landslides, erosion and other consequences such as drought, disrupted water cycles, infertile soil, in the end the forest has the possibility of extinction (Wirmayanti et al., 2021).

Various strategies to control forest damage due to illegal logging have been carried out. Control is carried out by direct handling to improve damaged conditions while preventing the recurrence of forest damage. Some of them are forest rehabilitation, greening, reforestation, and other rehabilitation efforts (Sundra, 2017). In the same context Muhammad et al. (2023) said that greening is an effort to protect land by planting trees with several objectives including preventing flooding, maintaining groundwater quality, protecting animals, reducing air pollution, climate control, reducing dust particles, preventing the greenhouse effect, supplying natural fertilizers, preventing erosion, and maintaining springs. The advancement of science, especially in the field of mathematics, is certainly a very helpful tool in solving problems in everyday life.

Mathematical modeling is the process of transforming a problem in everyday life into a mathematical form so that it is easy to find a solution (Wulandari et al., 2016). There have been many studies on mathematical models. The main focus of this research, that is, by adding the factor of forest resource damage control strategy to the model that has been introduced by previous researchers, including Suci et al. (2014) and Mohamad et al. (2019). The difference in this study lies in adding the variable of forest resource damage control strategy and ignoring the industrial factor and the fire factor. The next goal is to provide insight into problems in everyday life that can be solved using mathematical models. The variables used in this study are divided into three, which are the density of forest resources (H), population density (P), and control strategies for forest resource damage due to illegal logging (R). The model developed will then be observed how the dynamics of each condition of the equilibrium point obtained.

B. RESEARCH METHOD

The method used in this research is a literature study, that is, by conducting a literature review on the theory that supports the problem at hand. The literature review used was obtained from journals, articles, and reference books related to forest damage and its control as well as mathematical models. The stages carried out in making this research model, which can be seen systematically in the flow chart in Figure 1 below:

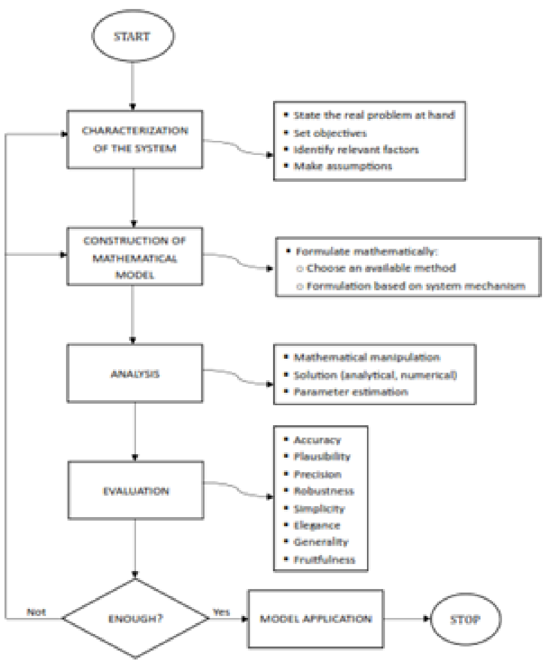

Figure 1. Stages in the model construction process

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C. RESULT AND DISCUSSION

1. Mathematical Modelling

The mathematical model developed in this study is a mathematical model and simulation of a strategy to control damage to forest resources due to illegal logging. There are three classes used in this study forest resource density class (H), population density class (P), and forest resource damage control strategy class due to illegal logging (R). The assumptions used in the model, are as follows:

a. Growth of forest resources is limited by the environmental carrying capacity of forest resources.
b. The growth of population is limited by the carrying capacity of the environment to the population.
c. Damage to forest resources occurs due to an increasing population of people who cut down forests illegally.
d. Population density increases because of the forest resources that support their lives.
e. Damage to forest resources is reduced due to strategies to control damage to forest resources.
f. Birth and death rates in the population are equal.
g. There is no displacement in the population.
h. Forest resource damage control strategies are improved due to government support.

Based on the assumptions, a flowchart of the mathematical model of the strategy to control damage to forest resources due to illegal logging is obtained, as presented in Figure 2.

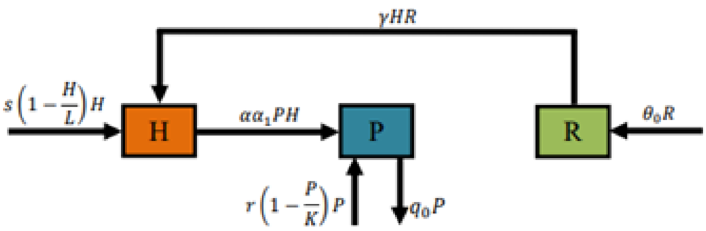

Figure 2. Flowchart of the mathematical model of the strategy to control damage to forest resources due to illegal logging

The rate of change in the density class of forest resources over time is influenced by the natural growth rate of forest resources (s) which is limited by the carrying capacity of the environment (L), then will decrease due to the increase in the density of the population of people who carry out illegal logging (α), but will increase due to the strategy of controlling damage to forest resources due to illegal logging (γ).

$$\frac{dH}{dt} = s\left(1 - \frac{H}{L}\right)H - \alpha PH + \gamma HR$$

The rate of change in the population density class over time is influenced by the natural growth rate of the population (r) which is limited by the carrying capacity of the environment (K), then it will increase due to the population density of people who cut down forests illegally (α) and the available forest resources that can support the fulfillment of their needs (α_1), but will decrease due to the population experiencing natural death (q_0).

$$\frac{dP}{dt} = r\left(1 - \frac{P}{K}\right)P + \alpha\alpha_1 PH - q_0 P$$

The rate of change in the class of forest resource damage control strategies due to illegal logging over time is influenced by the increase in forest resource damage control strategies and then will decrease due to the rate of forest resource density due to the forest resource damage control strategy due to illegal logging (γ).

$$\frac{dR}{dt} = \theta_0 R - \gamma HR$$

Based on Figure 2, a mathematical model of the forest resource damage control strategy is obtained in the form of a system of differential equations as follows:

$$\frac{dH}{dt} = s\left(1 - \frac{H}{L}\right)H - \alpha PH + \gamma HR \quad \frac{dP}{dt} = r\left(1 - \frac{P}{K}\right)P + \alpha\alpha_1 PH - q_0 P \quad \frac{dR}{dt} = \theta_0 R - \gamma HR \tag{1}$$

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2. Determination of the Equilibrium Point

The obtained equilibrium point is determined by the condition (Suddin et al., 2021):

$$\frac{dH}{dt} = 0, \ \frac{dP}{dt} = 0 \text{ and } \frac{dR}{dt} = 0$$

thus producing two equilibrium points, are:

a. Disturbance Free Equilibrium Point

The first equilibrium point is the disturbance free equilibrium point (E_1). The disturbance-free equilibrium point (E_1) means that there is no damage to forest resources. This means that there is no damage to forest resources due to illegal logging, and there is no strategy to control damage to forest resources due to illegal logging, so it is obtained:

$$E_1 = (H = L, P = 0, R = 0)$$

b. Equilibrium Point with Disturbance

The second equilibrium point is the equilibrium point with disturbance (E_2). The equilibrium point with disturbance (E_2) means that there is damage to forest resources. This is because of an increasing population of people who cut down forests illegally but there is no strategy to control damage to forest resources due to illegal logging, so that it is obtained:

$$E_2 = \left(H^* = \frac{L\left(sr + K\alpha q_0 - K\alpha r\right)}{LK\alpha^2\alpha_1 + rs}, P^* = \frac{Ks\left(\alpha\alpha_1 L - q_0 + r\right)}{LK\alpha^2\alpha_1 + rs}, R^* = 0 \right)$$

Because of point E_2 is not necessarily positive, it will exist if the following conditions are fulfilled:

- $r\left(s - K\alpha\right) + K\alpha q_0 > 0$ then $s > K\alpha$
- $\alpha\alpha_1 L + r > q_0$

Equilibrium point three, that is: the equilibrium point with disturbance (E_3). The equilibrium point with disturbance (E_3) means that there is damage to forest resources caused by the increasing population of people who cut down forests illegally and there is a strategy to control damage to forest resources due to illegal logging, so that it is obtained:

$$E_3 = \left(H^{**} = \frac{\theta_0}{\gamma}, P^{**} = \frac{K\left(\alpha\alpha_1\theta_0 - q_0\gamma + r\gamma\right)}{r\gamma}, R^{**} = \frac{-L\gamma rs + r\theta_0 s + \alpha^2\alpha_1\theta_0 KL - \alpha\gamma q_0 KL + \alpha r\gamma KL}{r\gamma^2 L} \right)$$

Because of point E_3 is not necessarily positive, it will exist if the following conditions are fulfilled:

- $r > q_0$
- $L\gamma < \theta_0$

3. Analysis of Equilibrium Point Stability

Suppose the right-hand side of Equation (1) can be written in the form of:

$$\begin{aligned} f_1\left(H, P, R\right) &= s\left(1 - \tfrac{H}{L}\right) H - \alpha PH + \gamma HR \\ f_2\left(H, P, R\right) &= r\left(1 - \tfrac{P}{K}\right) P + \alpha\alpha_1 PH - q_0 P \\ f_3\left(H, P, R\right) &= \theta_0 R - \gamma HR \end{aligned}$$

Determination of stability around the equilibrium point is firstly done linearization of Equation (1), then obtained Jacobian matrix (Suddin et al., 2021):

$$\begin{aligned} J &= \begin{bmatrix} \frac{\partial f_1}{\partial H} & \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial R} & \frac{\partial f_2}{\partial H} & \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial R} & \frac{\partial f_3}{\partial H} & \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial R} \end{bmatrix} \\ &= \begin{bmatrix} s - \frac{2sH}{L} - \alpha P + \gamma R & -\alpha H & \gamma H & \alpha\alpha_1 P & r - \frac{2rP}{K} + \alpha\alpha_1 H - q_0 & 0 & -\gamma R & 0 & \theta_0 - \gamma H \end{bmatrix} \end{aligned}$$

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a. Stability Analysis of Disturbance Free Equilibrium Point (E_1)

Substitute the disturbance free equilibrium point $E_1 = (H = L, P = 0, R = 0)$ to the Jacobi matrix, resulting in:

$$J_{E_1} = [-s \; -\alpha L \; \gamma L \; 0 \; r + \alpha\alpha_1 L - q_0 \; 0 \; 0 \; 0 \; \theta_0 - \gamma L \,]$$

then, the next step is to find the characteristic equation with the following steps: $det(\lambda I - J_{E_1}) = 0$ (Abi et al., 2023).

$$\Longleftrightarrow |[\lambda \; 0 \; 0 \; 0 \; \lambda \; 0 \; 0 \; 0 \; \lambda \,] - [-s \; -\alpha L \; \gamma L \; 0 \; r + \alpha\alpha_1 L - q_0 \; 0 \; 0 \; 0 \; \theta_0 - \gamma L \,]| = 0 \tag{2}$$

$$\Longleftrightarrow |[\lambda + s \; \alpha L \; -\gamma L \; 0 \; \lambda - (r + \alpha\alpha_1 L - q_0) \; 0 \; 0 \; 0 \; \lambda - (\theta_0 - \gamma L) \,]| = 0 \tag{3}$$

by applying Sarrus' rule in Masdiana et al. (2022), the determinant of the matrix in Equation (3) can be calculated as follows:

$$\begin{gathered}
|\lambda + s \; \alpha L \; -\gamma L \; 0 \; \lambda - (r + \alpha\alpha_1 L - q_0) \; 0 \; 0 \; 0 \; \lambda - (\theta_0 - \gamma L) \,| \; \lambda + s \; \alpha L \; 0 \; \lambda - (r + \alpha\alpha_1 L - q_0) \; 0 \; 0 \quad = 0 \\
\Longleftrightarrow \; ((\lambda \; + \; s)\,(\lambda - (r + \alpha\alpha_1 L - q_0))\,(\lambda - (\theta_0 - \gamma L)) + (\alpha L)\,(0)\,(0) + (-\,\gamma L)\,(0)\,(0) \\
-\,((-\gamma L)\,(\lambda - (r + \alpha\alpha_1 L - q_0))\,(0) + (\lambda \; + \; s)\,(0)\,(0) + (\alpha L)\,(0)\;(\lambda - (\theta_0 - \gamma L)) = 0 \\
\Longleftrightarrow \; ((\lambda \; + \; s))\,(\lambda - (r + \alpha\alpha_1 L - q_0))\,(\lambda - (\theta_0 - \gamma L)) = \; 0
\end{gathered}$$

so that obtained eigenvalues (Ndii, 2018):

$$\lambda_1 = -s, \; \lambda_2 = r + \alpha\alpha_1 L - q_0, \text{ and } \lambda_3 = \theta_0 - \gamma L$$

Based on eigenvalues λ_1, λ_2, and λ_3 obtained, it is recognised that $\lambda_1 < 0$, point E_1 is stable if $\lambda_2 < 0$ and $\lambda_3 < 0$ or $r + \alpha\alpha_1 L < q_0$ and $\theta_0 < \gamma L$ (Munaqib, 2021). If $r + \alpha\alpha_1 L > q_0$ and $\theta_0 > \gamma L$ disturbance free equilibrium point (E_1) is unstable.

b. Stability analysis of equilibrium point with disturbance $(E_2 \; and \; E_3)$

1) Stability analysis of equilibrium point with disturbance $E_2 = (H^*, P^*, R^*)$:

$$E_2 = \left(H^* = \frac{L\,(sr + K\alpha q_0 - K\alpha r)}{LK\alpha^2\alpha_1 + rs}, P^* = \frac{Ks\,(\alpha\alpha_1 L - q_0 + r)}{LK\alpha^2\alpha_1 + rs}, R^* = 0 \right)$$

substitute the equilibrium point with disturbance E_2 to the Jacobi matrix, producing:

$$J_{E2} = \left[s - \frac{2sH^*}{L} - \alpha P^* \; -\alpha H^* \; \gamma H^* \; \alpha\alpha_1 P^* \; r - \frac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0 \; 0 \; 0 \; 0 \; \theta_0 - \gamma H^* \right]$$

then, the next step is to find the characteristic equation with the following steps: $det(\lambda I - J_{E2}\,) = 0$ (Abi et al., 2023).

$$\begin{gathered}
\Longleftrightarrow \left| [\lambda \; 0 \; 0 \; 0 \; \lambda \; 0 \; 0 \; 0 \; \lambda \,] - \left[s - \tfrac{2sH^*}{L} - \alpha P^* \; -\alpha H^* \; \gamma H^* \; \alpha\alpha_1 P^* \; r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0 \; 0 \; 0 \; 0 \; \theta_0 - \gamma H^* \right] \right| = 0 \\
\Longleftrightarrow \left| \lambda - \left(s - \tfrac{2sH^*}{L} - \alpha P^* \right) \; \alpha H^* \; -\gamma H^* \; -\alpha\alpha_1 P^* \; \lambda - \left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0 \right) \; 0 \; 0 \; 0 \; \lambda - (\theta_0 - \gamma H^*) \right| = 0
\end{gathered}$$

by applying Sarrus' rule in Masdiana et al. (2022), the determinant of the matrix in Equation (6) can be calculated as follows:

$$\begin{aligned}
&\Longleftrightarrow \left(\lambda - \left(s - \tfrac{2sH^*}{L} - \alpha P^* \right) \right) \left(\lambda - \left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0 \right) \right) (\lambda - (\theta_0 - \gamma H^*)) \; + \; (\alpha H^*\,)\,(0)\,(0) \\
&+ \; (-\gamma H^*\,)\,(-\alpha\alpha_1 P^*)\,(0) - \; (-\,\gamma H^*) \left(\lambda - \left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0 \right) \right) (0) \\
&- \left(\lambda - \left(s - \tfrac{2sH^*}{L} - \alpha P^* \right) \right) (0)\,(0) - \; (\alpha H^*)\,(\alpha\alpha_1 P^*\,)\,(\,\lambda \; - \; \theta 0 \; - \; \gamma H^*\,)\,(\lambda - (\theta_0 - \gamma H^*)) \\
&= \; 0
\end{aligned}$$

$$\begin{aligned}
&\Longleftrightarrow \left(\lambda - \left(s - \tfrac{2sH^*}{L} - \alpha P^* \right) \right) \left(\lambda - \left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0 \right) \right) (\lambda - (\theta_0 - \gamma H^*)) \\
&- \; (\alpha H^*)\,(\alpha\alpha_1 P^*\,)\,(\,\lambda \; - \; \theta_0 \; - \; \gamma H^*\,) = \; 0
\end{aligned}$$

for example,

$$a = s - \frac{2sH^*}{L} - \alpha P^*,\ b = r - \frac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0,\ c = \theta_0 - \gamma H^*,\ d = \alpha H^*,\ e = \alpha\alpha_1 P^*$$

the characteristic equation above can be simplified into:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

then

$$\begin{aligned}
&[(\lambda - a)(\lambda - b)(\lambda - c) + (d)(e)(\lambda - c)] = 0\\
&\Leftrightarrow \lambda^3 - (a+b+c)\lambda^2 + (ab+bc+ac)\lambda - abc + de\lambda - cde = 0\\
&\Leftrightarrow\ \lambda^3 - (a+b+c)\lambda^2 + (ab+bc+ac+de)\lambda - abc - cde = 0\\
&\Leftrightarrow\ \lambda^3 - (a+b+c)\lambda^2 + (ab+bc+ac+de)\lambda - c(ab+de) = 0
\end{aligned}$$

Equation (7) can be simplified into the following characteristic equation

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

by

$$\begin{aligned}
a_1 &= -\left[\left(s - \tfrac{2sH^*}{L} - \alpha P^*\right) + \left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0\right) + (\theta_0 - \gamma H^*)\right]\\
a_2 &= \left(s - \tfrac{2sH^*}{L} - \alpha P^*\right)\left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0\right) + \left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0\right)(\theta_0 - \gamma H^*)\\
&\quad + \left(s - \tfrac{2sH^*}{L} - \alpha P^*\right)(\theta_0 - \gamma H^*) + (\alpha H^*)(\alpha\alpha_1 P^*)\\
a_3 &= -(\theta_0 - \gamma H^*)\left(\left(s - \tfrac{2sH^*}{L} - \alpha P^*\right)\left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0\right) + (-\alpha H^*)(\alpha\alpha_1 P^*)\right)
\end{aligned}$$

According to the Routh-Hurwitz criterion, the equilibrium point of E_2 will be stable if it satisfies the criteria: $a_1 > 0,\ a_2 > 0,\ \ a_3 > 0 and a_1 a_2 - a_3 > 0$ (Aakash et al., 2024).

1. Criterion check: $a_1 > 0$.

$$\begin{aligned}
a_1 &= -\left[\left(s - \tfrac{2sH^*}{L} - \alpha P^*\right) + \left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0\right) + (\theta_0 - \gamma H^*)\right]\\
&= -\left[\left(-\tfrac{s(K\alpha q_0 - K\alpha r + rs)}{LK\alpha^2\alpha_1 + rs}\right) + \left(-\tfrac{rs(\alpha\alpha_1 L - q_0 + r)}{LK\alpha^2\alpha_1 + rs}\right)\right.\\
&\quad \left. + \left(-\tfrac{L(K\alpha q_0 - K\alpha r + rs)\gamma + \theta_0(LK\alpha^2\alpha_1 + rs)}{LK\alpha^2\alpha_1 + rs}\right)\right]\\
&= \tfrac{1}{LK\alpha^2\alpha_1 + rs}\left[s(K\alpha q_0 + r(-K\alpha + s)) + rs(L\alpha\alpha_1 - q_0 + r)\right.\\
&\quad \left. + L(K\alpha q_0 + r(-K\alpha + s))\gamma + \theta_0(LK\alpha^2\alpha_1 + rs)\right]
\end{aligned}$$

   Because of $K\alpha q_0 r(-K\alpha + s) > 0$ and $L\alpha\alpha_1 - q_0 + r > 0$, then it is obtained $a_1 > 0$.

2. Criterion check: $a_2 > 0$.

$$\begin{aligned}
a_2 &= \left(s - \tfrac{2sH^*}{L} - \alpha P^*\right)\left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0\right)\\
&\quad + \left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0\right)(\theta_0 - \gamma H^*)\\
&\quad + \left(s - \tfrac{2sH^*}{L} - \alpha P^*\right)(\theta_0 - \gamma H^*) + (\alpha H^*)(\alpha\alpha_1 P^*)\\
&= \tfrac{1}{(LK\alpha^2\alpha_1 + rs)^2}\left[rs^2(K\alpha q_0 + r(-K\alpha + s))(\alpha\alpha_1 L - q_0 + r)\right.\\
&\quad + (rs(\alpha\alpha_1 L - q_0 + r))\left(LK\gamma\alpha q_0 + L\gamma r(-K\alpha + s) + LK\alpha^2\alpha_1\theta_0 + rs\theta_0\right)\\
&\quad + (K\alpha q_0 s + rs(-K\alpha + s))\left(LK\gamma\alpha q_0 + L\gamma r(-K\alpha + s) + LK\alpha^2\alpha_1\theta_0 + rs\theta_0\right)\\
&\quad \left. + LK\alpha^2\alpha_1 s(K\alpha q_0 + r(-K\alpha + s))(\alpha\alpha_1 L - q_0 + r)\right]
\end{aligned}$$

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3. Criterion check: $a_3 > 0$.

$$
\begin{aligned}
a_3 &= -\left(\theta_0 - \gamma H^*\right)\left(\left(s - \tfrac{2sH^*}{L} - \alpha P^*\right)\left(r - \tfrac{2rP^*}{K} + \alpha\alpha_1 H^* - q_0\right)\right.\\
&\quad \left. + \left(\alpha H^*\right)\left(\alpha\alpha_1 P^*\right)\right)\\
&= \tfrac{1}{(LK\alpha^2\alpha_1 + rs)^3}\left(LK\gamma\alpha q_0 + L\gamma r(-K\alpha + s) + LK\alpha^2\alpha_1\theta_0 + rs\theta_0\right)\\
&\quad \times\Big[\left(K\alpha q_0 s + rs\left(-K\alpha + s\right)\right)\left(rs\left(\alpha\alpha_1 L - q_0 + r\right)\right)\\
&\quad + \left(KL\alpha^2 q_0 + \alpha Lr\left(-K\alpha + s\right)\right)\left(KL\alpha^2{\alpha_1}^2 s - \alpha\alpha_1 K s q_0 + \alpha\alpha_1 K s r\right)\Big]
\end{aligned}
$$

Because of $-K\alpha + s > 0$ and$L\alpha\alpha_1 - q_0 + r > 0$, then it is obtained $a_3 > 0$.

4. Criterion check: $a_1 a_2 - a_3 > 0$.

$$
\begin{aligned}
a_1 a_2 - a_3 &= \tfrac{1}{(LK\alpha^2\alpha_1 + rs)^3}\left[\{s(K\alpha q_0 + r(-K\alpha + s)) + rs(L\alpha\alpha_1 - q_0 + r)\right.\\
&\quad + L(K\alpha q_0 + r(-K\alpha + s))\gamma + \theta_0(LK\alpha^2\alpha_1 + rs)\Big\}\\
&\quad \times\Big\{rs^2(K\alpha q_0 + r(-K\alpha + s))(\alpha\alpha_1 L - q_0 + r)\\
&\quad + (rs\alpha\alpha_1 L + rs(-q_0 + r))(LK\gamma\alpha q_0 + L\gamma r(-K\alpha + s) + LK\alpha^2\alpha_1\theta_0 + rs\theta_0)\\
&\quad + (K\alpha q_0 s + rs(-K\alpha + s))(LK\gamma\alpha q_0 + L\gamma r(-K\alpha + s) + LK\alpha^2\alpha_1\theta_0 + rs\theta_0)\\
&\quad + LK\alpha^2\alpha_1 s(K\alpha q_0 + r(-K\alpha + s))(\alpha\alpha_1 L - q_0 + r)\Big\}\\
&\quad - (LK\gamma\alpha q_0 + L\gamma r(-K\alpha + s) + LK\alpha^2\alpha_1\theta_0 + rs\theta_0)\\
&\quad \times\Big\{(K\alpha q_0 s + rs(-K\alpha + s))(rs(\alpha\alpha_1 L - q_0 + r))\\
&\quad \left. + (KL\alpha^2 q_0 + \alpha Lr(-K\alpha + s))(\alpha\alpha_1 K s(\alpha\alpha_1 L - q_0 + r))\Big\}\right]
\end{aligned}
$$

for example: $X = K\alpha q_0 + r(-K\alpha + s)$

$$Y = L\alpha\alpha_1 - q_0 + r$$
$$Z = L\gamma\left(K\alpha q_0 + r(-K\alpha + s)\right) + \theta_0\left(LK\alpha^2\alpha_1 + rs\right)$$

then;

$$
\begin{aligned}
&= \tfrac{1}{(LK\alpha^2\alpha_1 + rs)^3}\Big[\{sX + rsY + Z\}\{rs^2XY + rsYZ + sXZ + LK\alpha^2\alpha_1 sXY\}\\
&\quad - Z\{sXrsY + \alpha LX\alpha\alpha_1 KsY\}\Big]\\
&= \tfrac{1}{(LK\alpha^2\alpha_1 + rs)^3}\Big[rs^3X^2Y + rs^2XYZ + s^2X^2Z + LK\alpha^2\alpha_1 s^2X^2Y + r^2s^3XY^2 + r^2s^2Y^2Z\\
&\quad + rs^2XYZ + LK\alpha^2\alpha_1 s^2rXY^2 + rs^2XYZ + rsYZ^2 + sXZ^2 + LK\alpha^2\alpha_1 sXYZ - XYZs(LK\alpha^2\alpha_1 + rs)\Big]\\
&= \tfrac{1}{(LK\alpha^2\alpha_1 + rs)^3}\Big[rs^3X^2Y + s^2X^2Z + LK\alpha^2\alpha_1 s^2X^2Y + r^2s^3XY^2 + r^2s^2Y^2Z\\
&\quad + rs^2XYZ + LK\alpha^2\alpha_1 s^2rXY^2 + rs^2XYZ + rsYZ^2 + sXZ^2 + LK\alpha^2\alpha_1 sXYZ + rs^2XYZ\\
&\quad - XYZs(LK\alpha^2\alpha_1 + rs)\Big]\\
&= \tfrac{1}{(LK\alpha^2\alpha_1 + rs)^3}\Big[rs^3X^2Y + s^2X^2Z + LK\alpha^2\alpha_1 s^2X^2Y + r^2s^3XY^2 + r^2s^2Y^2Z\\
&\quad + rs^2XYZ + LK\alpha^2\alpha_1 s^2rXY^2 + rs^2XYZ + rsYZ^2 + sXZ^2 + sXYZ(LK\alpha^2\alpha_1 + rs)\\
&\quad - XYZs(LK\alpha^2\alpha_1 + rs)\Big]\\
&= \tfrac{1}{(LK\alpha^2\alpha_1 + rs)^3}\Big[rs^3X^2Y + s^2X^2Z + LK\alpha^2\alpha_1 s^2X^2Y + r^2s^3XY^2 + r^2s^2Y^2Z\\
&\quad + rs^2XYZ + LK\alpha^2\alpha_1 s^2rXY^2 + rs^2XYZ + rsYZ^2 + sXZ^2\Big]
\end{aligned}
$$

Because of $-K\alpha + s > 0$ and $L\alpha\alpha_1 - q_0 + r > 0$, then it is obtained $a_1 a_2 - a_3 > 0$.

Based on the examination of the Routh-Hurwitz criteria, the condition of: $a_1 > 0,\ a_2 > 0,\ a_3 > 0$ and $a_1 a_2 - a_3 > 0$ is satisfied, then the equilibrium point with disturbance E_2 is local asymptotically stable.

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2) Stability Analysis of of equilibrium point with disturbance (E_3):

$$E_3 = \left(H^{**} = \frac{\theta_0}{\gamma}, P^{**} = \frac{K(\alpha\alpha_1\theta_0 - q_0\gamma + r\gamma)}{r\gamma},\ R^{**} = \frac{-L\gamma rs + r\theta_0 s + \alpha^2\alpha_1\theta_0 KL - \alpha\gamma q_0 KL + \alpha r\gamma KL}{r\gamma^2 L} \right)$$

substitute the equilibrium point with disturbance E_3 to Jacobi matrix, resulting in:

$$J_{E_3} = \left[s - \frac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**} \ \ -\alpha H^{**}\ \gamma H^{**}\ \alpha\alpha_1 P^{**}\ r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\ 0\ \ -\gamma R^{**}\ 0\ \theta_0 - \gamma H^{**} \right]$$

by

$$a_1 = s - \frac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**},\ a_2 = \alpha H^{**},\ a_3 = \gamma H^{**}, a_4 = \alpha\alpha_1 P^{**},$$
$$a_5 = r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0, a_6 = \gamma R^{**}, a_7 = \theta_0 - \gamma H^{**}.$$

then, the next step is to find the characteristic equation with the following steps: $det(\lambda I - J_{E_3}) = 0$ (Abi et al., 2023).

$$\iff |[\lambda\ 0\ 0\ 0\ \lambda\ 0\ 0\ 0\ \lambda\,] - [a_1\ \ -a_2\ a_3\ a_4\ a_5\ 0\ \ -a_6\ 0\ a_7\,]| = 0$$
$$\iff |[\lambda - a_1\ a_2\ \ -a_3\ -a_4\ \lambda - a_5\ 0\ a_6\ 0\ \lambda - a_7\,]| = 0$$

by applying Sarrus' rule in Masdiana et al. (2022), the determinant of the matrix in Equation (9) can be calculated as follows:

$$|\lambda - a_1\ a_2\ -a_3\ \ -a_4\ \lambda - a_5\ 0\ a_6\ 0\ \lambda - a_7|\ \ \lambda - a_1\ a_2\ -a_4\ \lambda - a_5\ a_6\ 0$$
$$\iff (\lambda - (s - 2sH^{**}L - \alpha P^{**} + \gamma R^{**}))\left(\lambda - \left(r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right)\right)$$
$$\cdot (\lambda - (\theta_0 - \gamma H^{**})) + (\alpha H^{**})(0)(\gamma R^{**}) + (-\gamma H^{**})(-\alpha\alpha_1 P^{**})(0)$$
$$-(-\gamma H^{**})\left(\lambda - \left(r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right)\right)(\gamma R^{**})$$
$$-\left(\lambda - \left(s - \frac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**}\right)\right)(0)(0)$$
$$-(\alpha H^{**})(-\alpha\alpha_1 P^{**})(\lambda - (\theta_0 - \gamma H^{**})) = 0$$
$$\iff (\lambda - (s - 2sH^{**}L - \alpha P^{**} + \gamma R^{**}))\left(\lambda - \left(r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right)\right)$$
$$\cdot (\lambda - (\theta_0 - \gamma H^{**})) + (\gamma H^{**})\left(\lambda - \left(r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right)\right)(\gamma R^{**})$$
$$+(\alpha H^{**})(\alpha\alpha_1 P^{**})(\lambda - (\theta_0 - \gamma H^{**})) = 0$$

for example,

$$a = s - \frac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**},\ b = r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0,\ c = \theta_0 - \gamma H^{**},\ d = \gamma H^{**},\ e = \gamma R^{**},\ f = \alpha H^{**},$$
$$g = \alpha\alpha_1 P^{**}$$

then

$$[(\lambda - a)(\lambda - b)(\lambda - c) + (d)(e)(\lambda - b) + (f)(g)(\lambda - c)] = 0$$
$$\iff \lambda^3 - (a + b + c)\lambda^2 + (ab + bc + ac)\lambda - abc + de\lambda - bde + fg\lambda - cfg = 0$$
$$\iff \lambda^3 - (a + b + c)\lambda^2 + (ab + bc + ac + de + fg)\lambda - abc - bde - cfg = 0$$

Equation (10) can be simplified into the following characteristic equation:

by

$$\begin{aligned}
a_1 &= -\left[\left(s - \frac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**}\right) + \left(r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right) + (\theta_0 - \gamma H^{**})\right], \\
a_2 &= \left(s - \frac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**}\right)\left(r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right) \\
&\quad + \left(r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right)(\theta_0 - \gamma H^{**}) \\
&\quad + \left(s - \frac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**}\right)(\theta_0 - \gamma H^{**}) \\
&\quad + (\gamma H^{**})(\gamma R^{**}) + (\alpha H^{**})(\alpha\alpha_1 P^{**}), \\
a_3 &= -\left(s - \frac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**}\right)\left(r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right)(\theta_0 - \gamma H^{**}) \\
&\quad - \left(r - \frac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right)(\gamma H^{**})(\gamma R^{**}) \\
&\quad - (\theta_0 - \gamma H^{**})(\alpha H^{**})(\alpha\alpha_1 P^{**})
\end{aligned}$$

According to the Routh-Hurwitz criterion, the equilibrium point of the E_3 will be stable if it satisfies the criteria: $a_1 > 0,\ a_2 > 0,\ \ a_3 > 0$ and $a_1 a_2 - a_3 > 0$ (Aakash et al., 2024).

1. Criterion check: $a_1 > 0$.

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$$
\begin{aligned}
a_1 &= -\left[\left(s - \tfrac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**}\right) + \left(r - \tfrac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right) + (\theta_0 - \gamma H^{**})\right] \\
&= \tfrac{s\theta_0 + L\alpha\alpha_1\theta_0 + L\gamma(-q_0+r)}{\gamma L}
\end{aligned}
$$

Because of $-q_0 + r > 0$, then it is obtained $a_1 > 0$.

2. Criterion check: $a_2 > 0$.

$$
\begin{aligned}
a_2 &= \left(s - \tfrac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**}\right)\left(r - \tfrac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right) \\
&+ \left(r - \tfrac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right)(\theta_0 - \gamma H^{**}) \\
&+ \left(s - \tfrac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**}\right)(\theta_0 - \gamma H^{**}) \\
&+ (\gamma H^{**})(\gamma R^{**}) + (\alpha H^{**})(\alpha\alpha_1 P^{**}) \\
&= \tfrac{1}{r\gamma^2 L}\left[rs\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r)) + \theta_0\gamma\left(rs(-L\gamma+\theta_0) + \alpha^2\alpha_1\theta_0 KL + \alpha\gamma KL(-q_0+r)\right)\right. \\
&\left. + KL\alpha^2\alpha_1\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r))\right]
\end{aligned}
$$

Because of $-L\gamma + \theta_0 > 0$ and $-q_0 + r > 0$, then it is obtained $a_2 > 0$.

3. Criterion check: $a_3 > 0$.

$$
\begin{aligned}
a_3 &= -\left(s - \tfrac{2sH^{**}}{L} - \alpha P^{**} + \gamma R^{**}\right)\left(r - \tfrac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right)(\theta_0 - \gamma H^{**}) \\
&- \left(r - \tfrac{2rP^{**}}{K} + \alpha\alpha_1 H^{**} - q_0\right)(\gamma H^{**})(\gamma R^{**}) \\
&- (\theta_0 - \gamma H^{**})(\alpha H^{**})(\alpha\alpha_1 P^{**}) \\
&= \tfrac{1}{r\gamma^2 L}\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r))\left(rs(-L\gamma+\theta_0) + \alpha^2\alpha_1\theta_0 KL + \alpha\gamma KL(-q_0+r)\right)
\end{aligned}
$$

Because of $-L\gamma + \theta_0 > 0$ and $-q_0 + r > 0$, then it is obtained $a_3 > 0$.

4. Criterion check: $a_1 a_2 - a_3 > 0$.

$$
\begin{aligned}
a_1 a_2 - a_3 &= \tfrac{1}{r\gamma^3 L^2}\left[(s\theta_0 + L\alpha\alpha_1\theta_0 + L\gamma(-q_0+r))\right. \\
&\times \{rs\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r)) + \theta_0\gamma(rs(-L\gamma+\theta_0) \\
&+ \alpha^2\alpha_1\theta_0 KL + \alpha\gamma KL(-q_0+r)\ ) \\
&\left. + KL\alpha^2\alpha_1\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r))\ \}\right] \\
&- \tfrac{1}{r\gamma^2 L}\left[\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r))\right. \\
&\left. \times\left(rs(-L\gamma+\theta_0) + \alpha^2\alpha_1\theta_0 KL + \alpha\gamma KL(-q_0+r)\right)\right] \\
&= \tfrac{1}{r\gamma^3 L^2}\left[(s\theta_0 + L\alpha\alpha_1\theta_0 + L\gamma(-q_0+r))\right. \\
&\times \{rs\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r)) + \theta_0\gamma(rs(-L\gamma+\theta_0) \\
&+ \alpha^2\alpha_1\theta_0 KL + \alpha\gamma KL(-q_0+r)\ ) \\
&+ KL\alpha^2\alpha_1\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r)) \\
&\left. - (L\gamma)\left(\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r))\left(rs(-L\gamma+\theta_0) + \alpha^2\alpha_1\theta_0 KL + \alpha\gamma KL(-q_0+r)\right)\right)\right]
\end{aligned}
$$

for example,

$X = s\theta_0 + L\alpha\alpha_1\theta_0 + L\gamma(-q_0+r),\ Y = \alpha\alpha_1\theta_0 + \gamma(-q_0+r),\ Z = rs(-L\gamma+\theta_0) + \alpha^2\alpha_1\theta_0 KL$
$+\alpha\gamma KL(-q_0+r)$

then

$$
\begin{aligned}
&= \tfrac{1}{r\gamma^3 L^2}\Bigg[(s\theta_0 + L\alpha\alpha_1\theta_0 + L\gamma(-q_0+r)) \\
&\times\Big\{rs\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r)) + \theta_0\gamma\left(rs(-L\gamma+\theta_0) + \alpha^2\alpha_1\theta_0 KL + \alpha\gamma KL(-q_0+r)\right) \\
&+ KL\alpha^2\alpha_1\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r))\Big\} \\
&- L\gamma\theta_0(\alpha\alpha_1\theta_0 + \gamma(-q_0+r))\left(rs(-L\gamma+\theta_0) + \alpha^2\alpha_1\theta_0 KL + \alpha\gamma KL(-q_0+r)\right)\Bigg]
\end{aligned}
$$

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$$= \frac{1}{r\gamma^3 L^2}\left[X\left\{rs\theta_0 Y + \theta_0\gamma Z + KL\alpha^2\alpha_1\theta_0 Y\right\} - L\gamma\theta_0 YZ\right]$$
$$= \frac{1}{r\gamma^3 L^2}\left[\theta_0 XY(rs + KL\alpha^2\alpha_1) + \theta_0\gamma Z(X - LY)\right]$$
$$= \frac{1}{r\gamma^3 L^2}\Big[\theta_0 XY(rs + KL\alpha^2\alpha_1) + \theta_0\gamma Z(s\theta_0 + L\alpha\alpha_1\theta_0 + L\gamma(-q_0 + r)$$
$$-L\alpha\alpha_1\theta_0 - L\gamma(-q_0 + r))\Big]$$
$$= \frac{1}{r\gamma^3 L^2}\left[\theta_0 XY(rs + KL\alpha^2\alpha_1) + s{\theta_0}^2\gamma Z\right]$$

Because of $-L\gamma + \theta_0 > 0$ and $-q_0 + r > 0$, then it is obtained $a_1 a_2 - a_3 > 0$.

Based on the examination of the Routh-Hurwitz criteria, the condition of: $a_1 > 0,\ a_2 > 0,\ a_3 > 0$ and $a_1 a_2 - a_3 > 0$ is satisfied, then the equilibrium point with disturbance E_3 is local asymptotically stable.

4. Numerical Simulation

Numerical simulations were conducted using Wolfram Mathematica 10.0. When simulating, what is considered is the condition of each equilibrium point. Two equilibrium points are obtained, which are: the disturbance free equilibrium point E_1 and equilibrium point with disturbance E_2 and E_3. The parameter values used to perform the simulation are presented in Table 1.

Table 1. Description and Values of the parameters used in the simulation of the mathematical model of forest resource damage control strategy due to illegal logging

Parameter	Description	Value	Source
s	Natural growth rate of forest resource	1	Mohamad et al. (2019)
r	Growth rate of resident population	0.3	Mohamad et al. (2019)
α	Rate of destruction of forest resources caused by the increasing population density of people who cut down forests illegally	0.19	Assumption
α_1	Average population using forest resources for their needs	1.04	Mohamad et al. (2019)
γ	Rate of forest resource density due to strategies to control damage to forest resources from illegal logging	0.5	Assumption
γ	Natural population mortality rate	0.2	Mohamad et al. (2019)
θ_0	Average strategy to control damage to forest resources by the government due to illegal logging	0.8	Assumption
K	Environmental carrying capacity of population	5	Mohamad et al. (2019)
L	Environmental carrying capacity of forest resources	6	Mohamad et al. (2019)

a. Simulation of the disturbance free equilibrium point (E_1)

Simulations are conducted to see the dynamics of the condition of the disturbance free equilibrium point E_1. The disturbance free equilibrium point E_1 in question is a condition where there is no damage to forest resources. This is because there is no resident population that logs forests illegally and there is also no strategy to control damage to forest resources due to illegal logging. At the time of observation, the initial value of the forest resource class (H) was 100%, while the initial value of the population density class (P) and the strategy to control damage to forest resources due to illegal logging was 0%. The initial value of each parameter can be seen in Table 1. The dynamics of each class are shown in Figures 3a, 3b, and 3c.

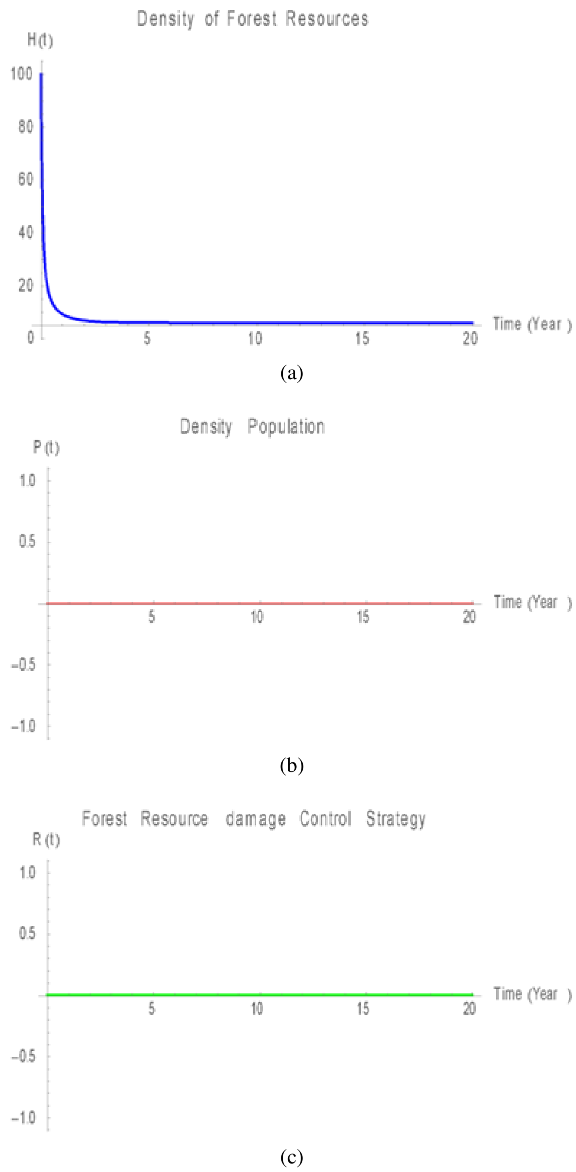

Figure 3. Simulation of system (1) at the disturbance free equilibrium point (E_1)

The disturbance free Equilibrium Point (E_1) of system (1) is E_1 (H, P, R)=(6,0,0). From Figures 3a, 3b and 3c, it can be seen that the density of forest resources (H) is decreasing and will stabilize at point 6, while the population density (P) and the strategy to control damage to forest resources due to illegal logging (R) are always stable at point 0. Therefore, if forest resources have not been affected by disturbances from population density and forest resource damage control strategies, the density of forest resources will be as high as the carrying capacity of the environment. Because if the forest is not used at all by the surrounding population, then there is no need for control strategies, the density of forest resources will be as high as the carrying capacity of the forest's environment, that is, animals, plants and water.

b. Simulation of the Equilibrium Point with Disturbance (E_2)

Simulations are conducted to see the dynamics of the equilibrium point with disturbance (E_2). The equilibrium point with disturbance (E_2) in question is the condition of forest resource damage. This is due to the increasing population of people who cut down forests illegally but there is no strategy to control damage to forest resources due to illegal logging. During the observation, the initial value of the forest resource class (H) was 80%, the initial value of the population density class (P) was 20% and the initial value of the forest resource damage control strategy class due to illegal logging was 0%. While the value of each parameter can be seen in Table 1. The dynamics of each class are shown in Figures 4a, 4b and 4c.

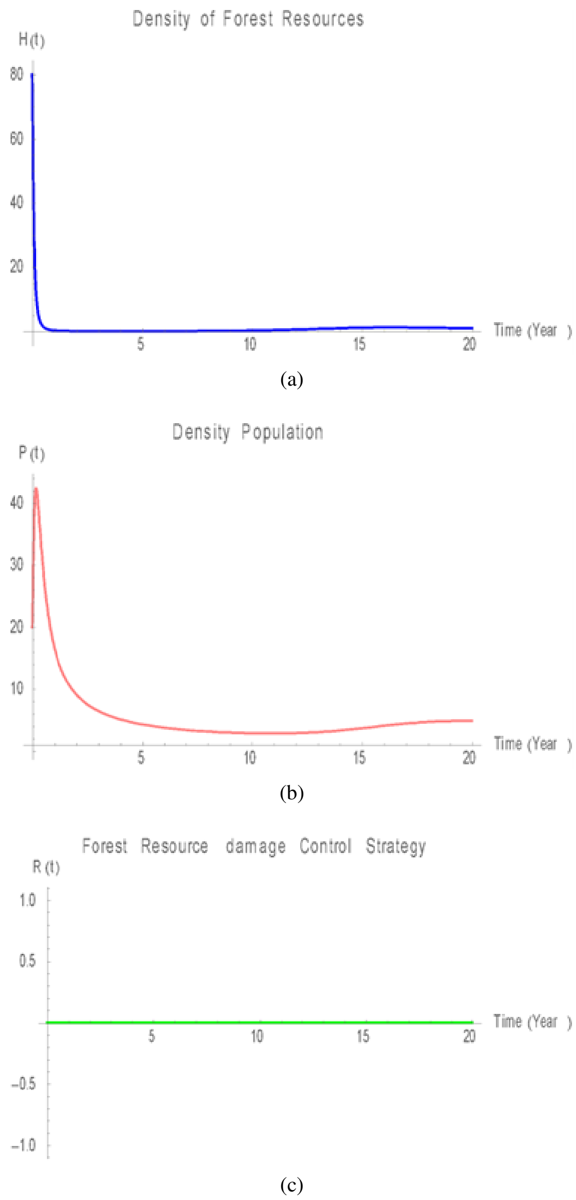

Figure 4. Simulation of system (1) at the equilibrium point with disturbance (E_2)

The equilibrium point with disturbance (E_2) of system (1) is $E_2\ (H^*,\ P^*,\ R^*) = (0.8623,\ 4.5067,\ 0)$. From Figures 4a, 4b, and 4c, it can be seen that the density of forest resources (H) has decreased and will stabilize at point 0.86234, then the population density (P) has increased then decreased and stabilized at point 4.5067. Meanwhile, the strategy to control damage to forest resources due to illegal logging (R) is always stable at point 0. Therefore, if forest resources have been affected by the disturbance of forest resource density while there is no strategy to control damage to forest resources, the density of forest resources will decrease. This is because if there is a surge in population density, which will result in many people using forest products for their needs, including expanding their living areas and also to create agricultural land for communities around the forest, so that forests will be slashed and burned. If there is no awareness from the community around the forest to preserve the forest and there is no attention from the government, then the density of forest resources will decrease and the forest will eventually become extinct and can cause many animals to die and the water cycle to be disrupted so that it can cause drought.

c. Simulation of the Equilibrium Point with Disturbance (E_3)

Simulations are conducted to see the dynamics of the condition of the equilibrium point with disturbance (E_3). The equilibrium point with disturbance (E_3) means that there is damage to forest resources caused by an increasing population

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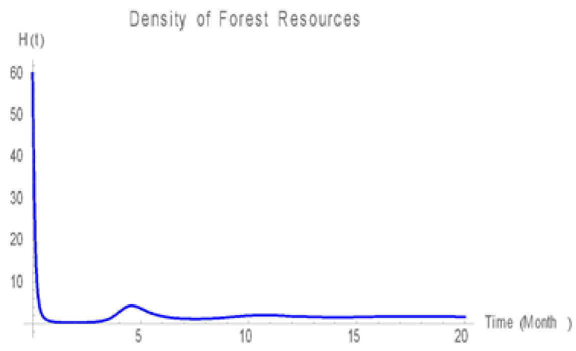
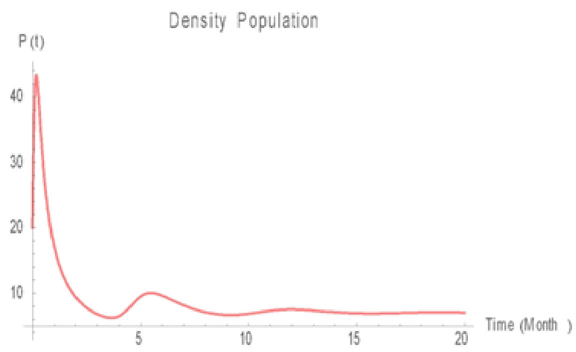
of people who cut down forests illegally and there is a strategy to control damage to forest resources due to illegal logging. During the observation, the initial value of the forest resource class (H) was 60%, the initial value of the population density class (P) was 20% and the initial value of the forest resource damage control strategy class due to illegal logging was 20%. The values of each parameter can be seen in Table 1. The dynamics of each class are shown in Figures 5a, 5b and 5c.



(a)



(b)

(c)

Figure 5. Simulation of system (1) at the equilibrium point with disturbance (E_3)

The equilibrium point with disturbance (E_3) of system (1) is $E_3\ (H^{**},\ P^{**},\ R^{**}) = (1.6,\ 6.936,\ 1.1690)$. From Figures 4a, 4b, and 4c, it shows that the density of forest resources (H) decreased, then increased and then decreased and stabilized at 1.6, then the population density (P) increased and decreased and stabilized at 6,936. While the strategy to control damage to forest resources due to illegal logging (R) also experienced an increase and decrease and stabilized at point 1.1690. Therefore, if forest resources have been affected by the disturbance of population density, then balanced with the existence of a forest resource damage control strategy, forest resources will increase. This is because if there is an increase in population so that the use of forest resources for the community around the forest is increasing, then balanced with the attention of the government and the community around the forest on forest resource damage control strategies, that is, by repairing damaged conditions while preventing the recurrence of forest damage by carrying out forest rehabilitation, reforestation, reforestation, and other rehabilitation efforts, then the density of forest resources will increase, resulting in animals, plants, and water being maintained so that they can overcome drought. This is in line with previous research that considers the factors of industrial activities and fires by Suci et al. (2014) and Mohamad et al. (2019). The difference is that the focus of this research is to pay attention to the strategy of controlling forest damage due to illegal logging.

D. CONCLUSION AND SUGGESTION

Based on the research and simulation results of the model, it can be concluded that taking into account the variable of forest resource damage control strategy due to illegal logging, the result shows that if the density of forest resources has been affected by the disturbance of population density around the forest, it is necessary to have a forest resource damage control strategy in order to compensate for the people around the forest who do a lot of illegal logging. In order to maintain the forest so that the forest does not quickly become extinct and can overcome drought, prevent flooding, maintain groundwater quality, protect animals, reduce air pollution, climate control, reduce dust particles, prevent the greenhouse effect, supply natural fertilizers, prevent erosion, and maintain springs. This research is expected to be a reference for developing further research in the field of mathematical modeling and simulation. This effort is made to expand understanding in the field of mathematical modeling and simulation.

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DECLARATIONS

The first author has a role and responsibility in designing research and submitting titles to the study program. Then after being approved for guidance, the second author is responsible with the first author for designing research from writing proposals, results seminars, submitting articles to journals then thesis trials. then the second author acts as corresponding author to make revisions until publication, the third author is responsible for guiding the first author from writing proposals, results seminars, to thesis trials.

Because this is the result of a student's final project, so there are no funds used in conducting this research.

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