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The NADI Mathematical Model on the Danger Level of the Bili-Bili Dam

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ABSTRACT

The research discusses the NADI mathematical model due to the overflow of the Bili-Bili dam, using secondary data obtained through online literature review by collecting various information related to the Bili-Bili Dam, starting from the Jeberang River Scheme, the chronology of floods, normal or dry conditions, and dam operation patterns. The aim of this study is to predict the level of danger of Bili-bili dam overflow over time, considering extreme weather factors and standard operating procedures performed by humans. The research uses analytical and computational methods. The study obtained the NADI mathematical model due to the overflow of the Bili-Bili dam, with two equilibrium points: (1) the equilibrium point free of disaster, (2) the disaster equilibrium point, and a basic disaster reproduction number of R0 = 1.219. This indicates that the water discharge from the dam is high and has an impact on the overflowing water for communities around the Jeneberang river. Therefore, it can be concluded that the NADI model can be used to simulate the Bili-bili dam process based on extreme weather and dam SOP, and predict the level of danger of Bili-bili dam overflow, which is also a novelty that has not been done in previous studies.

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A. INTRODUCTION

In the historical development of civilization in the world, there are many innovative ideas to help solve human problems. One of the problems that humans often face is natural disasters (Suwaryo and Yuwono, 2017), which have a negative impact on all aspects of human life. One of the most frequent natural disasters in Indonesia is flooding (Darmawan et al., 2017; Niode et al., 2016). Floods are the most frequent natural disasters which are around 40 percent of other natural disasters (Risman et al., 2016). The flood itself is one of the natural phenomena that always occurs in the rainy season. Flooding occurs due to high rainfall intensity which causes excess water that exceeds the capacity limit in an area. The flood plain area is a lowland area on the side of the river which has a very gentle and relatively flat elevation. So, to reduce this, a reservoir is built and large excess runoff during the rainy season can be accommodated in the reservoir. South Sulawesi with the capital city of Makassar is famous for the Bili-Bili reservoir which is located in Gowa Regency, South Sulawesi.

The reservoir was built with a main building height of 73 m and a length of 750 m. The catchment area of the reservoir is 384.40 km^2 with a storage capacity of 375 million m^3. The Bili-Bili reservoir is the confluence of the Jeneberang River and the Jenelata River. The Bili-Bili reservoir is multifunctional, namely as a flood control or reducing discharge (Achsan et al., 2015) where the discharge is 2200 m^3/s to 1200 m^3/s, the provision of raw water sources is 3300 liters/s, irrigation water services with an area of

23690 hectares of potential, Hydroelectric Power Plant (PLTA) with an installed capacity of 20.1 MW and as a tourism area. It is conceivable that the collapse of the Bili-Bili Dam could cause damage many times over. According to standard procedures, if the water level exceeds the normal limit, the dam's floodgates must be opened to prevent the dam from collapsing. The Regional Disaster Management Agency (BPBD) of South Sulawesi recorded 106 villages affected by the disaster in 61 sub-districts spread over 13 districts/cities. The 13 districts/cities include Jeneponto, Maros, Gowa, Makassar City, Soppeng, Wajo, Barru, Pangkep, Sidrap, Bantaeng, Takalar, Selayar, and Sinjai regencies. Therefore, it is important to develop a model to solve the problem of the overflow of the Bili-Bili dam.

Susceptible, Exposed, infectious, and recovered (SEIR) models have been used in various fields. Syafruddin Side, Alimuddin, and Alvioni Bani modified the SIR model (Side et al., 2018a) on the spread of Dengue Hemorrhagic Fever in Bone Regency. Stability analysis and numerical simulation of the SEIR model for the pandemic COVID-19 spread in Indonesia (Annas et al., 2020; Side et al., 2018c), SEIAS-SEI model on asymptomatic and superinfection malaria with imperfect vaccination (Maryam et al., 2021), analysis and solution of the SEIRS model for the Rubella transmission with vaccination effect using Runge-Kutta Method (Asri et al., 2021; Side et al., 2018a), analysis and simulation of mathematical model for typhus disease in Makassar (Anas et al., 2021) have been studied. However, these studies only focused on the model of the spread of the disease. Research that combines the SIR/SEIR model by involving the problem regarding factors faced by the Bili-Bili Reservoir has not been thoroughly explored. In this research, the combination of both the SIR/SEIR model and information on the Bili-Bili reservoir is used as the basis for developing a model to find a solution to the problem of the Jeneberang river overflow by developing the NADI mathematical model.

1. Fluid

Fluid (Risman et al., 2016), in physics, is a substance or subsystem that will continuously deform when exposed to a shear force (tangential force), even though the force is small. When there is an overflow of water across the crest of a spillway, flow contraction occurs both on the side walls of the spillway and around the pillars built on top of the spillway, so that hydraulically the effective width of a spillway will be smaller than the actual overall width of the spillway. The flow of water that crosses the spillway in question is always based on its effective width, which is the result of subtracting the actual width from the total number of contractions that occur in the flow of water passing through the spillway.

1.1. SEIR Mathematical Model

SEIR mathematical model (Side et al., 2018b) is a system of differential equations to calculate the number of model distributions or non-linear incidence rates in epidemiology. The global stability of catastrophic equilibrium is proved by using the general criteria for orbital stability of the periodic orbits associated with higher-dimensional nonlinear autonomous systems as well as the theory of competitive systems of differential equations. As far as development goes, this mathematical model that we refer to deals with the extent of disaster spread.

2. System Equilibrium Point

The equilibrium point is a state of a system that does not change with time. If the dynamics system is described in a differential equation, then the equilibrium point can be obtained by taking the first derivative which is equal to zero. In the Definition 1.1 (Side et al., 2021), point $\bar{x} \in R^n$ is called the equilibrium point from $\dot{x} = f(x)$ if $f(x) = 0$, where

$$f(x) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ f_2(x_1, x_2, \ldots, x_n) \\ \vdots f_n(x_1, x_2, \ldots, x_n) \end{bmatrix}$$

3. Stability Analysis of Equilibrium Point

The linear differential equation system of order n is given as follows:

$$x_1 = Ax + f_i(x_1, x_2, \ldots, x_n) \qquad (*)$$

where $i = 1, 2, \ldots, n$

The first step to get a solution to equation (*) is to find the equilibrium point. Suppose we obtained the equilibrium point $(x_1^*, x_2^*, \ldots, x_n^*)$, then the next step is to find the Jacobian matrix.

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Let $G_i(x_1, x_2, \ldots, x_n) = Ax + f_i(x_1, x_2, \ldots, x_n)$, the Jacobian matrix is

$$A = \begin{bmatrix} \frac{\partial G_1}{\partial x_1} & \cdots & \frac{\partial G_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial G_n}{\partial x_1} & \cdots & \frac{\partial G_n}{\partial x_n} \end{bmatrix}$$

The next step is the substitution of the equilibrium point in the Jacobian matrix to obtain the linear system as follows:

$$\dot{u}_i = A_{(x_1^*, x_2^*, \ldots, x_n^*)} u$$

Determination of the stability of the equilibrium point is obtained by looking at the eigenvalues λ_i with $i = 1, 2, \ldots, n$ as follows:

$$\det(\lambda I - A) = 0$$

In general, the stability of the equilibrium point has two behaviors, namely:

1. Stable if
   (a) $Re(\lambda_i) < 0$ for every i, or
   (b) There are $Re(\lambda_j) = 0$ for any j and $Re(\lambda_j) < 0$ for every ij.
2. Unstable if there is at least one i so that $Re(\lambda_i) > 0$ (Side et al., 2021)

4. Basic Reproductive Number

The basic reproduction number can be found using the next-generation matrix method. This matrix is formed by taking into account the positive and negative parts of the population transmission rate, namely the exposed and infected populations. The formula for determining the number reproduction base is given in the equation $K = F'(V')^{-1}$ (Side et al., 2021)

In general, the basic reproduction number R_0 has three possibilities, namely:
If $R_0 < 1$ then the probability of overflow at the Bili-Bili dam is low.
If $R_0 = 1$, then the Bili-Bili dam will be stable.
If $R_0 > 1$, then the probability of overflow of the Bili-Bili dam is high.

B. RESEARCH METHOD

This research is a theoretical and applied research study that is reviewing the literature on mathematical modeling related to the overflow of the Bili-Bili dam. The data used in this study is secondary data obtained from the literature that has been previously studied so that the resulting 4 stages in the dam are produced due to the volume of water in the dam, namely fluid sample $N(0)$ is 2000 m^3/s; score fluid sample $Ad(0)$ is 2500 m^3/s; score fluid sample $Ds(0)$ is 1426 m^3/s; score fluid sample $I(0)$ is 600 m^3/s, and sample W is maximum model fluid.

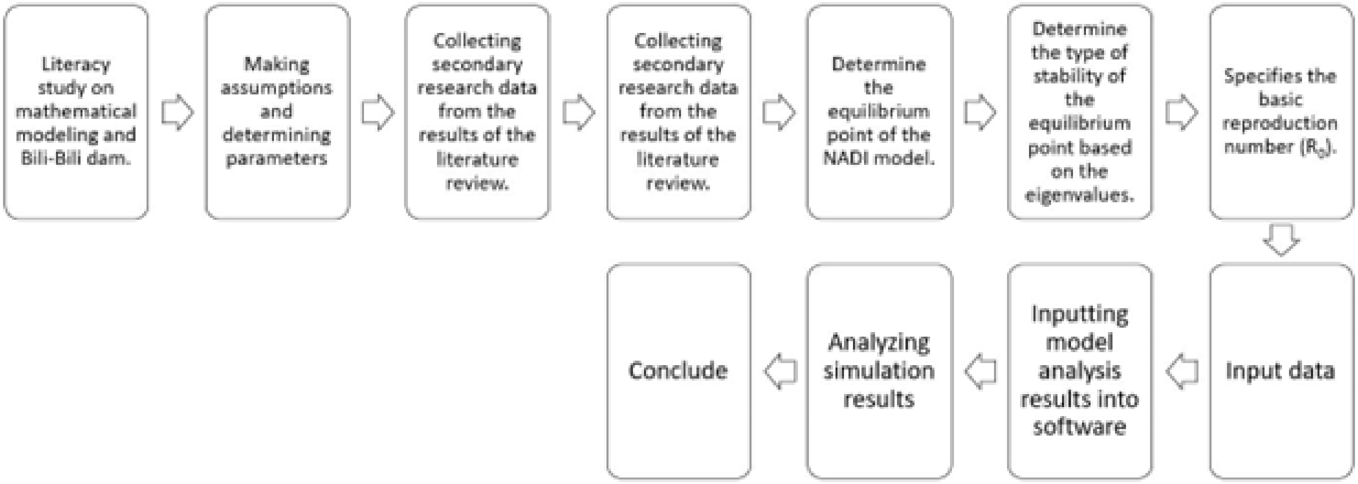

Figure 1. Research conducting flowchart

The steps taken in this research are as follows:

1. Literature Review
   (a) Literacy study on mathematical modeling and Bili-Bili dam.
   (b) Making assumptions and determining parameters
   (c) Collecting secondary research data from the results of the literature review.
2. Develop a mathematical model:
   (a) Building a NADI mathematical model
   (b) Determine the equilibrium point of the NADI model.
   (c) Determine the type of stability of the equilibrium point based on the eigenvalues.
   (d) Specifies the basic reproduction number (R_0).
3. Simulating the NADI mathematical model using the Maple 17 application:
   (a) Input data,
   (b) Inputting model analysis results into software,
   (c) Analyzing simulation results,
   (d) Conclude.

C. RESULTS AND DISCUSSION

1. NADI Mathematical Model of Overflow from Bili-Bili dam

In this research, the population is divided into four classified classes: (1) f-class (N) that is Normal Functional Flow or the flow of the Jeneberang i.e. 2000 m^3/s with a maximum of 2240 m^3/s; (2) f-class (AD) namely Alert-Danger Incubation or flow rate upstream river or Bili- Bili dam i.e. 2500 m^3/s with a maximum of 3199 m^3/s; (3) f-class (Ds) i.e. Disaster Pre Incubation or flow rate Bili-Bili Dam Spillway door i.e. 1427 m^3/s with maximum 1800 m^3/s; (4) f-class (I) namely Infection Recovery or Jenelata River flow rate i.e. 600 m^3/s with a maximum of 987 m^3/s. The maximum limit meant here is limit shelter from a variable, where if pass the limit, can be assumed the overflow happened and causes disaster.

There are several assumptions used in creating models, namely:

1. The population must have a system control rate.
2. The data rate of deep-water utilization at Bili- Bili dam is considered constant.
3. The data rate comply the fluid rule
4. The data entered into f-class (N) are Jeneberang River, Malino River, and Bontomanai. River as well as raining around downstream of the Jeneberang River.
5. The data entered into f-class (AD) from f-class (N) is the fluid flow of the Jeneberang River, Malino River, and Bontomanai. River fluids join rain above as well as around downstream of the Jeneberang River going to the estuary Bili-Bili dam
6. The data entered into f-class (AD) are rain over the estuary of the Bili-Bili Dam and annual sedimentation due to landslides

and other sedimentation factors.

7. The data entered into the f-class (Ds) from the f-class (AD) is the estuary fluid flow to the Spillway reservoir.
8. The data entered into the f-class (I) from the f-class (Ds) is the flow of the Spillway reservoir fluid to the Jenelata River when the Spillway door is opened.
9. The assumption of water utilization in the Bili-Bili dam is to make an artificial waterfall as a control gate to an artificial lake that functions as a water reservoir in Gowa Regency or create a new flow from the dam with a storage capacity of 1000 cubic meters per second to the Makassar Strait.

The developed SEIR model schema becomes a NADI mathematical model can be seen in Figure 2 below.

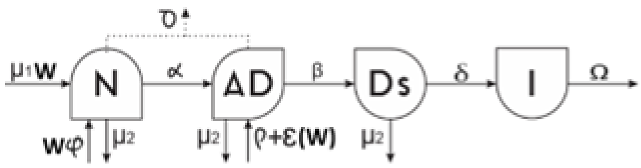

Figure 2. Schematic of the SEIR Model that has been developed become NADI model

Figure 2 can also be interpreted in the mathematical model which is an equality nonlinear differential like the following:

$$\frac{dn}{dt} = \mu W + \varphi W - (\alpha + \mu_2 + O)n \tag{1}$$

$$\frac{dds}{dt} = (\alpha)n + (\rho + \varepsilon)W - (\beta + \mu_2 + O)ad \tag{2}$$

$$\frac{dad}{dt} = (\beta)ad - (\delta + \mu_2)ds \tag{3}$$

$$\frac{di}{dt} = (\delta)ds - (\Omega)i \tag{4}$$

Table 1. Variables and parameters in the NADI model

No	Variables/Parameters	Information
1	n	Flow data Jeneberang River fluid
2	ad	Flow data fluid Bili-Bili dam
3	ds	Flow data Bili-Bili Dam Spillway fluid
4	i	Flow data Jenelata River fluid
5	μ_1	Fluid data rate enter to f-class (N)
6	μ_2	Fluid data rate community use of water
7	α	Rate fluid f-class (N) to f-class (AD) data transfer
8	β	Rate fluid f-class (AD) data transfer to f-class (Ds)
9	δ	Rate fluid data transfer f-class (Ds) to f-class (I)
10	O	Fluid outflow rate data solution
11	φ	Fluid data inflow rate to f-class (N)
12	$\rho + \varepsilon$	Fluid data inflow rate to f-class (AD)
13	Ω	Fluid data rate disposal to Makassar Strait

Where the data W is the fluid highest model.

2. Disaster NADI Model Analysis on overflow Bili-Bili dam

2.1. Equilibrium Point Analysis

The first step in determining point equilibrium free disaster and point equilibrium disaster is by simplifying equation (1)-(4) by dividing with W, that is $\left(N = \dfrac{n}{W}, Ad = \dfrac{ad}{W}, Ds = \dfrac{ds}{W}, I = \dfrac{i}{W}\right)$, so that we get equation (5)-(8)

$$\frac{dN}{dt} = \mu_1 + \varphi - (\alpha + \mu_2 + O)N \tag{5}$$

$$\frac{dAd}{dt} = (\alpha)N + (\rho + \varepsilon) - (\beta + \mu_2 + O)Ad \tag{6}$$

$$\frac{dds}{dt} = (\beta)Ad + (\delta + \mu_2)Ds \tag{7}$$

$$\frac{di}{dt} = (\delta)Ds + (\Omega)I \tag{8}$$

To determine point equilibrium free disaster and point equilibrium disaster, every equation in equation (5)-(8) must be equal to zero, that is, $\left(\frac{dS}{dt}, \frac{dAd}{dt}, \frac{dDs}{dt}, \frac{dI}{dt}\right) = (0,0,0,0)$, so that we get equation (9)-(12)

$$\mu_1 + \varphi - (\alpha + \mu_2 + O)N = 0 \tag{9}$$

$$(\alpha)N + (\rho + \varepsilon) - (\beta + \mu_2 + O)Ad = 0 \tag{10}$$

$$(\beta)Ad - (\delta + \mu_2)Ds = 0 \tag{11}$$

$$(\delta)Ds - (\Omega)I = 0 \tag{12}$$

Furthermore, using a simple substitution method, the values of S, Ad, Ds, and I will be determined for the disaster-free equilibrium point and the disaster equilibrium point. The disaster-free equilibrium point is the state where no occur deployment disaster overflow the Bili-Bili dam so that $Ad = 0$. By doing a little algebraic manipulation on equations (9)-(12) then we obtained equations (13)-(16) as follows:

$$N = \frac{\mu_1 + \varphi}{\alpha + \mu_2 + O} \tag{13}$$

$$Ad = \frac{\alpha + \rho + \varepsilon}{\beta + \mu_2 + O} \tag{14}$$

$$Ds = \frac{\beta}{\delta + \mu_2} \tag{15}$$

$$I = \frac{\delta}{\Omega} \tag{16}$$

By substituting each equation (13)-(16) by first determining the value $Ad = 0$ then the disaster-free equilibrium point is obtained as follows:

$$(N, Ad, Ds, I) = \left(\frac{\mu_1 + \varphi}{\alpha + \mu_2 + O}, 0, 0, 0\right) \tag{17}$$

Next, in the same way, by substituting equation (13)-(16) then the disaster equilibrium point is obtained as follows:

$$(N, Ad, Ds, I) = \begin{pmatrix} \dfrac{\mu_1 + \varphi}{\alpha + \mu_2 + O} \\ \dfrac{(\mu_1 + \varphi)(\alpha + \rho + \epsilon)}{(\alpha + \mu_2 + O)(\beta + \mu_2 + O)} \\ \dfrac{(\mu_1 + \varphi)(\alpha + \rho + \epsilon)(\beta)}{(\alpha + \mu_2 + O)(\beta + \mu_2 + O)(\delta + \mu_2)} \\ \dfrac{(\mu_1 + \varphi)(\alpha + \rho + \epsilon)(\beta)(\delta + \varphi)}{(\alpha + \mu_2 + O)(\beta + \mu_2 + O)(\delta + \mu_2)(\Omega)} \end{pmatrix} \tag{18}$$

2.2. Basic Reproduction Number

The basic reproduction number could be found using the next-generation matrix method. This matrix is formed by taking into account the positive and negative parts of speed fluid from along the waterway within the Bili-Bili dam area. The formula for determining the basic reproduction number is given in equation (19)

$$R = F' \cdot (V')^{-1} \tag{19}$$

Based on equation (6), then

$$\begin{aligned} \frac{dN}{dt} &= \mu_1 + \varphi - (\alpha + \mu_2 + O)N \\ \frac{dAd}{dt} &= (\alpha)N + (\rho + \varepsilon) - (\beta + \mu_2 + O)Ad \end{aligned}$$

So that obtained

$$F = \begin{bmatrix} \mu_1 + \varphi \\ 0 \end{bmatrix}, F' = \begin{bmatrix} 0 & \mu_1 + \varphi \\ 0 & 0 \end{bmatrix} \tag{20}$$

$$V = \begin{bmatrix} \alpha + \mu_2 + O \\ (\beta + \mu_2 + O)Ad - (\alpha)N + (\rho + \varepsilon) \end{bmatrix}, V' = \begin{bmatrix} \alpha + \mu_2 + O & 0 \\ -\alpha & \beta + \mu_2 + O \end{bmatrix} \tag{21}$$

So, we get the inverse matrix of equation (21) as follows:

$$(V')^{-1} = \begin{bmatrix} \dfrac{1}{\alpha + \mu_2 + O} & 0 \\ \dfrac{\alpha}{(\alpha + \mu_2 + O)(\beta + \mu_2 + O)} & \dfrac{1}{\beta + \mu_2 + O} \end{bmatrix} \tag{22}$$

The eigenvalue of matrix R is determined based on equation (19) as follows:

$$\begin{aligned} R &= \begin{bmatrix} 0 & \mu_1 + \varphi \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{1}{\alpha + \mu_2 + O} & 0 \\ \dfrac{\alpha}{(\alpha + \mu_2 + O)(\beta + \mu_2 + O)} & \dfrac{1}{\beta + \mu_2 + O} \end{bmatrix} \\ R &= \begin{bmatrix} \dfrac{\alpha(\mu_1 + \varphi)}{(\alpha + \mu_2 + O)(\beta + \mu_2 + O)} & \dfrac{\mu_1 + \varphi}{\beta + \mu_2 + O} \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{23}$$

After obtaining matrix R in equation (23), then the Eigenvalue is found with the formula $\det(\lambda I - R) = 0$, where I is matrix identity. The basic reproduction number is determined based on the highest eigenvalue (λ).

$$|\lambda I - J| = \left| \left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \dfrac{\alpha(\mu_1 + \varphi)}{(\alpha + \mu_2 + O)(\beta + \mu_2 + O)} & \dfrac{\mu_1 + \varphi}{\beta + \mu_2 + O} \\ 0 & 0 \end{bmatrix} \right) \right| = 0 \tag{24}$$

So that we obtained the eigenvalue based on equation (24) as follows:

$$\lambda_1 = \frac{\alpha(\mu_1 + \varphi)}{(\alpha + \mu_2 + O)(\beta + \mu_2 + O)} \quad \text{and} \quad \lambda_2$$

We get the highest eigenvalue which is $\lambda_1 = \dfrac{\alpha(\mu_1 + \varphi)}{(\alpha + \mu_2 + O)(\beta + \mu_2 + O)}$

The basic reproduction number based on equation (24) is given as follows:

$$R = \frac{\alpha(\mu_1 + \varphi)}{(\alpha + \mu_2 + O)(\beta + \mu_2 + O)} \tag{25}$$

Stability Analysis of Equilibrium Point

Based on equation (1) - (4), matrix Jacobian (J) can be formed as follows:

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$$J = \begin{bmatrix} -\mu_2 - \alpha - O & 0 & 0 & 0 \\ 0 & -\mu_2 - \beta - O & 0 & 0 \\ 0 & \beta & -\mu_2 - \delta & 0 \\ 0 & 0 & \delta & -\Omega \end{bmatrix} \tag{26}$$

Teorema
Disaster-free equilibrium point on the spread of Covid-19 is said stable if $R_0 \leq 1$ and not stable if $R_0 > 1$.
Proof.
Substitution of disaster-free equilibrium point to matrix J equation (26), so that we obtained new matrix as in equation (27).

$$J = \begin{bmatrix} -\mu_2 - \alpha - O & 0 & 0 & 0 \\ 0 & -\mu_2 - \beta - O & 0 & 0 \\ 0 & \beta & -\mu_2 - \delta & 0 \\ 0 & 0 & \delta & -\Omega \end{bmatrix} \tag{27}$$

Then the eigenvalue of the matrix in equation (27) is solved with a description as follows:

$$\begin{aligned} |\lambda I - J| &= \left| \left(\lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right) \right| - \begin{bmatrix} -\mu_2 - \alpha - O & 0 & 0 & 0 \\ 0 & -\mu_2 - \beta - O & 0 & 0 \\ 0 & \beta & -\mu_2 - \delta & 0 \\ 0 & 0 & \delta & -\Omega \end{bmatrix} \\ |\lambda I - J| &= \left| \lambda \begin{bmatrix} -\mu_2 - \alpha - O - \lambda & 0 & 0 & 0 \\ 0 & -\mu_2 - \beta - O - \lambda & 0 & 0 \\ 0 & \beta & -\mu_2 - \delta - \lambda & 0 \\ 0 & 0 & \delta & -\Omega - \lambda \end{bmatrix} \right| \end{aligned} \tag{28}$$

Furthermore, substitute Ad in equation (28) so that we obtained equation (29).

$$(\lambda + \mu)(\lambda + \mu_2 + \alpha + O)[\lambda^2((\mu_2 + \beta + O)(\delta + \mu))\lambda + (\mu_2 + \beta + O)(\delta + \mu) - R_0] = 0 \tag{29}$$

Based on the rule of Descartes sign, equation (29) will have the root of all negative if all marks on each of the tribes are positive. So, it can be concluded that a disaster-free equilibrium-free point is stable if $R0 \leq 1$ and not stable if $R_0 > 1$.

3. NADI Model Simulation in Disaster overflow from Bili-Bili dam

The simulation was conducted using Maple 17 software with a given score for each of the parameters. The parameter values are taken so that we obtained $R_0 = 1.2195 > 1$ which indicates that the event overflow and possibly collapse dam could happen. The score fluid sample $N(0)$ is $2000m^3/s$; the score fluid sample $Ad(0)$ is $2500m^3/s$; the score fluid sample $Ds(0)$ is $1426m^3/s$; score fluid sample $I(0)$ is $600m^3/s$; sample W is maximum model fluid.

Table 2. Parameter and Variable values in the Bili-Bili dam overflow disaster NADI model

Amateur par	Score	Source
μ_1	1	Assumption
μ_2	0.02	Assumption
α	1	Assumption
β	1	Assumption
δ	1	Assumption
O	0.8	Assumption
φ	1	Assumption
ρ	0.5	Assumption
Ω	1	Assumption
ε	1	Assumption

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3.1. Equilibrium Point of the NADI Model

The equilibrium point is determined using the NADI model set with the parameters for Bili-Bili that have already been defined. To determine the fixed point, the system of equations (1)-(4) is equated to zero and it is given in equations (9)-(12). By substituting the parameter values into equations (9)-(12), we get equations (30)-(33) as follows:

$$1 + 1 - (1.82)N = 0 \tag{30}$$

$$(1)N + (0.5 + 1) - (1.82)Ad = 0 \tag{31}$$

$$(1)Ad - (1.02)Ds = 0 \tag{32}$$

$$(1)Ds - (1)I = 0 \tag{33}$$

The system for the model in equations (30)-(33) is solved by using Maple 17 software. The values for the model's equilibrium points are:

$$(N, Ad, Ds, I) = (1.82; 1.82; 1.02; 1).$$

These equilibrium points explain that the potential for the Jeneberang river flow and the Bili-Bili dam to overflow is 182%, the potential for the Bili-Bili dam spillway flow is 102% or close to normal conditions, the Jenelata river flow potential to overflow is 100% or in under normal conditions. The most potential solution to be given is the Jeneberang river which has the greatest potential to trigger the overflow of the Bili-Bili dam.

3.2. Stability of the NADI Model of Overflow from the Bili-Bili dam

By using equations (30)-(33) and the specified parameter values, the NADI model is converted into a Jacobian matrix (34)-(37) to find the eigenvalues λ.

$$\frac{dN}{dt} = 1 + 1 - (1.82)N = N(n, dd, ds, i) \tag{34}$$

$$\frac{dAd}{dt} = (1)N + (0.5 + 1) - (1.82)Ad = Ad(n, dd, ds, i) \tag{35}$$

$$\frac{dDs}{dt} = (1)Ad - (1.02)Ds = Ds(n, dd, ds, i) \tag{36}$$

$$\frac{dI}{dt} = (1)Ds - (1)I = I(n, dd, ds, i) \tag{37}$$

To find the eigenvalues λ, solve the $|A - \lambda I| = 0$ as follows:

$$\begin{bmatrix} (-\mu_2 - \alpha - O)(N) - \lambda & 0 & 0 & 0 \\ 0 & (-\mu_2 - \alpha - O)(Ad) - \lambda & 0 & 0 \\ 0 & 0\beta & (-\mu_2 - \alpha - O)(Ds) - \lambda & 0 \\ 0 & 0 & \delta & -\Omega(I) - \lambda \end{bmatrix} = 0$$

Substituting equilibrium point values $(N, Ad, Ds, I) = (1.82; 1.82; 1.02; 1)$ so that we obtained:

$$\begin{bmatrix} (-\mu_2 - \alpha - O)(1.82) - \lambda & 0 & 0 & 0 \\ 0 & (-\mu_2 - \alpha - O)(1.82) - \lambda & 0 & 0 \\ 0 & 0\beta & (-\mu_2 - \alpha - O)(1.02) - \lambda & 0 \\ 0 & 0 & \delta & -\Omega(I) - \lambda \end{bmatrix} = 0$$

By using Maple 17 software, eigenvalues $\lambda = -0.0108988$, $\lambda = -3.31$, $\lambda = -3.31$, $\lambda = -1.04$, and $\lambda = -1$ were obtained. Values λ obtained at the equilibrium point (1.82; 1.82; 1.02; 1) are real and had a negative sign. Referring to nature stability, the type of stability at this equilibrium point is asymptotically stable.

3.3. Simulation Results of the NADI Model with Time Delay for the spread of the Bili-Bili Dam Overflow in South Sulawesi

The simulation of the NADI model without Stage II water level control can be seen in Figure 3.

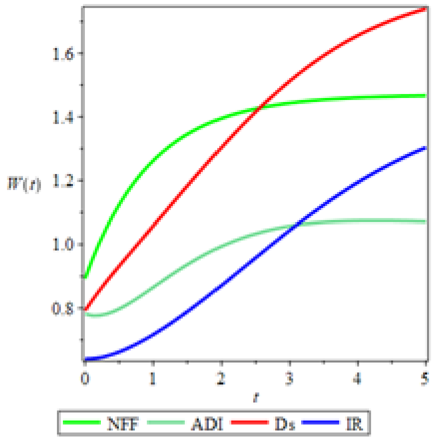

Figure 3. The plot of the NADI model without Water Level control Phase II

Figure 3 shows the model plot of the overflow of the Bili-Bili Dam. Plot (NFF) represents the overflowing Jeneberang River, Plot (ADI) represents the water level in the dam crossing the elevation limit (standby stage), Plot (Ds) represents the capacity of the spillway gate that reaches the limit (the dam that makes the water elevation in the dam increases faster and causes catastrophic collapse and will cause a greater impact than the January 22, 2020 event. Plot (IR) represents the Jenelata River overflowing and submerging residential houses around the Bili-Bili Dam and along the Jenelata River.

The simulation of the NADI model with Stage II water level control can be seen in Figure 4.

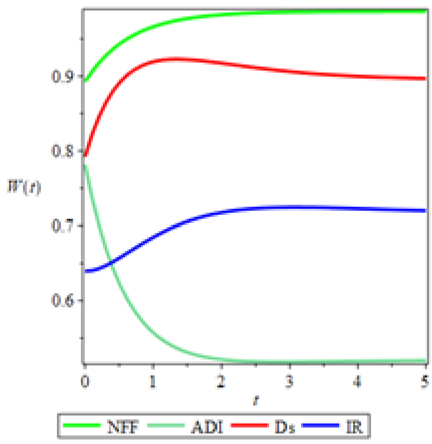

Figure 4. A plot of the NADI Model with Water Level Control Phase II

Figure 4 shows the model plot for the non-overflowing of the Bili-Bili Dam. Plot (NFF) represents the non-overflowing Jeneberang River, Plot (ADI) represents the water level in the dam that does not exceed the elevation limit (standby stage), Plot (Ds) represents the capacity of the spillway gate that does not reach the limit. Plot (IR) represents the Jenelata River which does not overflow and does not inundate residential houses around the Bili-Bili and along the Jenelata River.

Table 3. Results of the NADI Model for the Bili-Bili dam overflow disaster

No	Mathematical models	R_0 -value
1	The NADI model for the Bili-Bili dam overflow disaster	1.2195

Based on Table 3, the value of $R_0 = 1.2195$ is obtained for the NADI model of the Bili-Bili dam overflow disaster,

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which means that the probability of an overflow of the Bili-Bili dam is more than 100%. It can be interpreted that the Bili-Bili dam overflow disaster based on its equilibrium points tends to overflow.

4. Discussion

There are many studies related to measuring the rate characteristics of an object. For example, analysis and simulation of a mathematical model for typhus disease in Makassar (Anas et al., 2021); analysis and solution of the SEIRS model for the rubella transmission with vaccination effect using the Runge-Kutta method (Asri et al., 2021); SEIRI model analysis using the mathematical graph as a solution for Hepatitis B disease in Makassar (Side et al., 2021); numerical solution of SIRS model for dengue fever transmission in Makassar City with Runge-Kutta method (Asri et al., 2021); a mathematical model for the novel coronavirus epidemic in Wuhan, China (Yang et al., 2020); review on Covid-19 disease so far (Djalante et al., 2020; Shahreen et al., 2020; Sohrabi et al., 2020). However, these studies were only limited to the process of deployment disease. The mathematical model for measuring the level rate of the increase, decrease or equilibrium from the overflow Bili-Bili dam has not been explored yet.

Based on theoretical studies on mathematical modeling and theoretical studies related to dam technical and other supporting theories, the researchers succeeded in developing the NADI model as a model that can measure the rate of the overflow of the Bili-Bili dam.

The results of this study obtained a mathematical model for analyzing the rate stability of the Bili-Bili dam overflow case, two equilibrium points, namely the disaster-free equilibrium point and the disaster equilibrium point, and the reproduction number of the Bili-Bili dam overflow disaster. To reduce the overflow flow rate and the possibility of a dam collapse based on the NADI model simulation, the solutions offered are:

1. Reviewing the Standard Operating Procedure (SOP) in the process of opening the dam door so that the stored water flows out and there is no significant increase in water
2. Carry out reforestation so that the absorption area around the dam can suppress the increase in water level in the dam when there is prolonged rain;
3. Making the outflow of the dam the right solution as a process to balance the inflow, so that the outflow from the estuary does not have an impact on the surrounding community.

D. CONCLUSION AND SUGGESTION

NADI mathematical model produces two equilibrium points, namely the disaster-free equilibrium point and the disaster equilibrium point both of which are stable. After performing the model simulation, the plot of the model without the stage II controller and stage II controller is obtained and there is a difference in plot height on the same plot timeline in both model simulations. If no further action is taken, it will be estimated that the disaster caused by the Bili-Bili dam will always occur when the rainy season arrives or it rains heavily for 3-5 days in a row. In addition, there is a possibility that the dam will collapse so that it will have a greater impact than the previous overflow that occurred in January 2020. In contrast, when carrying out phase II control, it will be seen that there will be no overflow and the possibility of a collapse is almost impossible even though the rainy season arrives so that people no longer need to worry about rising water in the Bili-Bili reservoir.

The solution to the problem of an overflow or collapse of the Bili-Bili Dam that the researcher offers is to create an alternative outflow flow for the Bili-Bili Dam so that if the inflow flow in the dam increases significantly, the new outflow flow can balance the flow which keeps the flow stable. When the rainy season arrives, there will be no significant increase in the flow of the dam, and the community will no longer have to worry about a disaster.

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