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Optimum Control of SEIR Model on COVID-19 Spread with Delay Time and Vaccination Effect in South Sulawesi Province

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ABSTRACT

The increasing number of cases and the development of new variants of the Covid-19 virus globally including the territory of Indonesia, especially in the province of South Sulawesi are increasingly worrying and need to be prevented. Therefore, this study aims to develop a SEIR model on the spread of Covid-19 with vaccination control, optimal control analysis, stability analysis and numerical simulation of the SEIR model on the spread of Covid-19 in South Sulawesi. This study uses the SEIR epidemic model to predict the spread of Covid-19 in South Sulawesi Province with parameters such as birth rate, cure rate, mortality rate, interaction rate and vaccination. The SEIR model was chosen because it is one of the basic methods in the epidemiological model. The method used to build the model is a time delay model by considering the vaccination factor as a model parameter, model analysis using the next generation matrix method to determine the basic reproduction number and stability of the Covid-19 distribution model in South Sulawesi. Numerical model simulation using secondary data on the number of Covid-19 cases in South Sulawesi starting in 2021 which was obtained from the South Sulawesi Provincial Health Office. The results obtained are model analysis provides evidence of the existence of optimal control in the model. Based on the results obtained, it can also be seen that vaccination greatly influences the spread of Covid-19 in South Sulawesi, so that awareness is needed for the people of South Sulawesi to follow the government's recommendation to vaccinate to prevent or reduce the rate of transmission of Covid-19 in South Sulawesi.

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A. INTRODUCTION

The Covid-19 has occupied the world for almost two years since the World Health Organization (WHO) declared it a global pandemic in March 2020. This virus is a new and infectious disease caused by SARS-CoV-2 (Severe Acute Respiratory Syndrome Coronavirus 2) (WHO, 2021). According to the Center for Disease Control and Prevention (CDC), coronavirus spreads through direct and indirect contact, as well as through droplets, namely coughs and sneezes from nearby patients (Rundle et al., 2020). Acute respiratory illnesses such as cough, shortness of breath and fever are common symptoms and signs of Covid-19 infection, where severe symptoms often lead to pneumonia and even death (Kementerian Kesehatan RI, 2018).

The number of infection cases with the corona virus continues to increase in various parts of the world, both in the number of people infected, deaths, and recoveries. Each country also has its own policies to curb the spread of the virus in its territory. The following are the latest developments in several countries regarding the new type of coronavirus (Annas et al., 2020). According to data from Johns Hopkins University, as of March 23, 2020, the total number of Covid-19 cases worldwide reached 331,273, with

14,450 deaths, and 97,847 patients declared cured. The highest number of cases is recorded in China, namely 81,397, followed by Italy with 59,138, and the US with 33,073. In terms of death rates, the highest number is in Italy, with 5,476. This number exceeds the death toll that occurred in China, which was 3,265 while the highest cure rate is in China, with 72,362 patients. Good developments in the number of infections, deaths and recoveries for certain guidelines continue to be reported from various countries.

The increasing number and the development of new variants of this virus worldwide, including Indonesian territory, have turned the hospital health service facilities where patients are treated into a high-risk place for the spread of Covid-19, both for health workers, visitors, and patients. Health workers who are the front-line service provider to these patients are at high risk of contracting the virus from mild symptoms to death (Wahyuni and Kurniawidjaja, 2022). In an effort to prevent the spread of Covid-19, the Ministry of Health issued a Circular, Numbered HK.02.01/MENKES/199/2020, which is an appeal that all sectors of society must heed. In addition to government advisors, it is also necessary to pay attention to factors affecting COVID-19 compliance, including attitudes, motivation, knowledge, age, education, social environment, and available facility (Tanto and Handayani, 2022).

Mathematical modeling of SIR, SIRS, SEIR, and SEIRS for transmission of diseases such as dengue fever, tuberculosis, diabetes, and HIV-AIDS was performed by (Rusliza and Budin, 2012), (Annas et al., 2021), (Apenteng and Ismail, 2017), (Diekmann et al., 2010), (Dontwi et al., 2014), (Egonmwan and Okuonghae, 2019), (Rangkuti et al., 2014), (Side et al., 2017), (Side et al., 2021), (Spencer et al., 2020), (Waziri et al., 2012), (Demirci et al., 2011), while that of COVID-19 was carried out by (Annas et al., 2020) (Side, 2015), namely modeling SEIR mathematics for the Indonesian region and SEIRV mathematical modeling in the Wuhan area, China by considering environmental factors. Furthermore, research on optimal control mathematical modeling has been carried out by (Syafruddin and Noorani, 2013) using a strategy of vaccinating susceptible and treating infected individuals. These studies have not included optimal control and time delay in analyzing the mathematical model used. Therefore, in this study, an analysis of the SEIR model of the spread of Covid-19 was carried out by adding an analysis of optimal control and delay time.

B. RESEARCH METHOD

Optimal Control Analysis SEIR mathematical modeling on the spread of Covid-19 with a time delay is employed in this study. The method used to build the model is the SEIR (Diekmann et al., 2010) which is carried out through the addition of optimal control in the analysis section in order to determine the basic reproduction number of the model by using the generation matrix method to find out the state of transmission of Covid-19 in South Sulawesi (Side et al., 2021). The data used in this study is in the form of data relating to the spread of Covid-19 in South Sulawesi starting in 2021 which was obtained from the South Sulawesi Provincial Health Office. Also, numerical simulation model and Maple software is used to predict the number of Covid-19 cases in South Sulawesi as a step to prevent the increase in the virus growth.

C. RESULTS AND DISCUSSION

1. SEIR Mathematical Model Formulation on the Spread of Covid-19 in South Sulawesi

The SEIR model of the spread of Covid-19 in South Sulawesi is divided into four subpopulations, namely Suspected, Exposed, Infected, and Recovered, who are susceptible to contracting the virus due to the interactions with previously infected individuals. Changes that occur in each human subpopulation in Covid-19 transmission cases by using the SEIR model with a time delay is interpreted as shown in Figure 1 below:

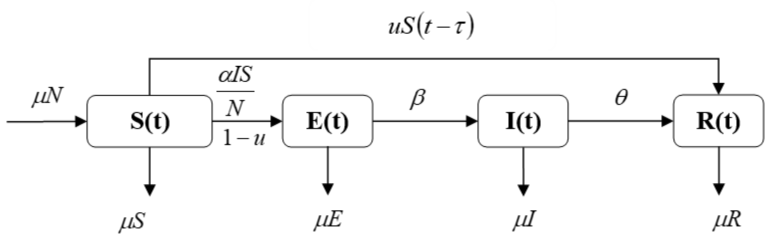

Figure 1. Schematic of the SEIR Model for the spread of Covid-19

The variables and parameters used in the SEIR mathematical model of the spread of Covid-19 with a time delay is seen in Table 1

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Table 1. The variables and parameters used in the SEIR mathematical model

Parameter	Description
N	Total population
α	The rate of movement from a human population susceptible to Covid-19 to a population that is symptomatic (exposed) but has not transmitted Covid-19due to interactions with infected populations
β	The rate of movement from a human population susceptible to Covid-19 to a population infected with Covid-19
θ	The rate of change from the human population infected with Covid-19 to the population recovering from Covid-19
S	Susceptible population infected with Covid-19
E	Symptomatic (exposed) population but have not transmitted Covid-19
I	Population infected with Covid-19
R	The population who have recovered from Covid-19
U	Control variables regarding the effectiveness of administering the Covid-19 vaccine
μ	The natural birth and death rates are assumed to be the same
t	Time
τ	Time delay

Based on the SEIR epidemic model SCHEME for the spread of Covid-19, the rate of change in the number of individuals in each subpopulation is interpreted as seen in equation (1) - (4).

$$\frac{dS}{dt} = \mu N - (\alpha I(1-v) + \mu)\, S(t-\tau) - vS(t-\tau) \tag{1}$$
$$\frac{dE}{dt} = \alpha IS(1-v) - (\beta + \mu)E \tag{2}$$
$$\frac{dI}{dt} = \beta E - (\theta + \mu)I \tag{3}$$
$$\frac{dR}{dt} = \theta I + vS(t-\tau) - \mu R \tag{4}$$

2. **Analysis of the SEIR Mathematical Model for the Spread of Covid-19 in South Sulawesi The SEIR Model Equilibrium Point**

To determine the disease-free and endemic equilibrium point, each equation in (1) - (4), is equated to zero, namely $\frac{dS}{dt} = 0$, $\frac{dE}{dt} = 0$, $\frac{dI}{dt} = 0$, dan $\frac{dR}{dt} = 0$, hence equations (5) - (8) are established:

$$0 = \mu N - (\alpha I(1-v) + \mu)\, S(t-\tau) - vS(t-\tau) \tag{5}$$
$$0 = \alpha IS(1-v) - (\beta - \mu)E \tag{6}$$
$$0 = \beta E - (\theta + \mu)I \tag{7}$$
$$0 = \theta I + vS(t-\tau) - \mu R \tag{8}$$

By using a simple substitution method, the values of S, E, I, and R is determined for the disease-free and the SEIR model endemic equilibrium point.

The disease-free equilibrium point is a condition where there is no spread of Covid-19 hence by performing a little algebraic manipulation in equations (5) - (8), the following equations (9) - (12) are obtained:

$$S(t-\tau) = \frac{\mu N}{\alpha I(1-v) + \mu + v} \tag{9}$$
$$E = \frac{\alpha IS(1-v)}{\beta + \mu} \tag{10}$$
$$I = \frac{\beta E}{\theta + \mu} \tag{11}$$
$$R = \frac{\theta I + vS}{\mu} \tag{12}$$

By substituting each equation in equation (9) - (12), the first value of $I = 0$ is determined. Subsequently, the disease-free equilibrium point of the SEIR model of the spread of Covid-19 in South Sulawesi is established as follows:

$$(S,E,I,R) = \left(\frac{\mu N}{u+v}, 0, 0, \frac{v\mu N}{\mu^2 + \mu v}\right) \tag{13}$$

In the same way, by substituting each equation in (9) - (12), the endemic equilibrium point value of the SEIR model is obtained as follows:

$$(S^*, E^*, I^*, R^*) = \begin{pmatrix} \dfrac{(\theta+\mu)(\beta+\mu)}{\alpha\beta(1-v)} \\ \dfrac{\alpha(1-v)S^*I^*}{\beta+\mu} \\ \dfrac{\alpha\beta\mu N(1-v) - (v+\mu)(\theta+\mu)(\beta+\mu)}{\alpha(1-v)(\theta+\mu)(\beta+\mu)} \\ \dfrac{\theta I^* + vS^*}{\mu} \end{pmatrix} \tag{14}$$

3. Analysis of the SEIR Mathematical Model for the Spread of Covid-19 in South Sulawesi The SEIR Model Equilibrium Point

The basic reproduction number is found by using the next generation matrix method, formed by considering the positive and negative parts of the transmission rate of the infected population, namely the exposed and infected. This formula for determining the basic reproduction number is seen in equation (15)

$$K = F' \cdot (V')^{-1} \tag{15}$$

Based on the system of equations (2) and (3), then:

$$\frac{dE}{dt} = \alpha IS(1-v) - (\beta+\mu)E$$
$$\frac{dI}{dt} = \beta E - (\theta+\mu)I$$

Therefore, it is obtained as

$$F = \begin{bmatrix} \alpha IS(1-v) \\ 0 \end{bmatrix}$$
$$F' = \begin{bmatrix} 0 & \alpha IS(1-v) \\ 0 & 0 \end{bmatrix} \tag{16}$$

$$V = \begin{bmatrix} (\beta+\mu)E \\ (\theta+\mu)I - \beta E \end{bmatrix}$$
$$V' = \begin{bmatrix} \beta+\mu & 0 \\ -\beta & \theta+\mu \end{bmatrix} \tag{17}$$

Then the inverse of the matrix equation (17) is established as

$$(V')^{-1} = \begin{bmatrix} \dfrac{1}{\beta+\mu} & 0 \\ \dfrac{\beta}{(\beta+\mu)(\theta+\mu)} & \dfrac{1}{\theta+\mu} \end{bmatrix} \tag{18}$$

Afterward, the eigenvalues of the K matrix is determined based on equation (15)

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$$K = \begin{bmatrix} 0 & \alpha IS(1-v) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\beta+\mu} & 0 \\ \frac{\beta}{(\beta+\mu)(\theta+\mu)} & \frac{1}{\theta+\mu} \end{bmatrix}$$

$$K = \begin{bmatrix} \frac{\alpha\beta S(1-v)}{(\beta+\mu)(\theta+\mu)} & \frac{\alpha S(1-v)}{\theta+\mu} \\ 0 & 0 \end{bmatrix} \tag{19}$$

After obtaining the K matrix in equation (19), the eigenvalues is gotten using the formula $\det(\lambda I - K) = 0$, where I is the identity matrix. The basic reproduction number is determined based on the largest (λ) eigenvalues.

$$|\lambda I - K| = \left| \left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{\alpha\beta S(1-v)}{(\beta+\mu)(\theta+\mu)} & \frac{\alpha S(1-v)}{\theta+\mu} \\ 0 & 0 \end{bmatrix} \right) \right| = 0 \tag{20}$$

Therefore the eigenvalues are established from equation (20), as follows

$$\lambda_1 = \frac{\alpha\beta S(1-v)}{(\beta+\mu)(\theta+\mu)}$$
$$\lambda_2 = 0$$

Also, the largest eigenvalue is obtained as $\frac{\alpha\beta S(1-v)}{(\beta+\mu)(\theta+\mu)}$
The basic reproduction number is determined after substituting the disease-free equilibrium point value as seen in equation (21)

$$R_0 = \frac{\alpha\beta\mu N(1-v)}{(\beta+\mu)(\theta+\mu)(\mu+v)} \tag{21}$$

4. Equilibrium Point Stability Analysis

Based on equation (1) - (4), the following Jacobian matrix (J) is formed

$$J = \begin{bmatrix} -(\alpha I(1-v)+\mu+v) & 0 & -\alpha S(1-v) & 0 \\ \alpha I(1-v) & -(\beta+\mu) & \alpha S(1-v) & 0 \\ 0 & \beta & -(\theta+\mu) & 0 \\ v & 0 & \theta & -\mu \end{bmatrix} \tag{22}$$

Theorem 1 *The disease-free equilibrium point for the mathematical model of the Covid-19 spread is said to be stable if* $R_0 \leq 1 \cdot 1$ *and unstable if* $R_0 > 1$.

Proof.
By substituting the disease-free equilibrium point into the J matrix of equation (22), a new matrix is acquired as seen in equation (23)

$$J = \begin{bmatrix} -(\mu+v) & 0 & -\alpha S(1-v) & 0 \\ 0 & -(\beta+\mu) & \alpha S(1-v) & 0 \\ 0 & \beta & -(\theta+\mu) & 0 \\ v & 0 & \theta & -\mu \end{bmatrix} \tag{23}$$

Then the eigenvalues is calculated by using equation matrix (23) with the following description:

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$$|\lambda I - J| = 0$$

$$|\lambda I - J| = \left| \left(\lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -(\mu+v) & 0 & -\alpha S(1-v) & 0 \\ 0 & -(\beta+\mu) & \alpha S(1-v) & 0 \\ 0 & \beta & -(\theta+\mu) & 0 \\ v & 0 & \theta & -\mu \end{bmatrix} \right) \right| = 0$$

$$|\lambda I - J| = \left| \begin{bmatrix} \lambda+(\mu+v) & 0 & -\alpha S(1-v) & 0 \\ 0 & \lambda+(\beta+\mu) & \alpha S(1-v) & 0 \\ 0 & \beta & \lambda+(\theta+\mu) & 0 \\ v & 0 & \theta & \lambda+\mu \end{bmatrix} \right| = 0 \tag{24}$$

By substituting S in equation (24), the following is obtained

$$(\lambda+\mu)(\lambda+\mu+u\cdot(t-\tau)^2)[\lambda^2+((\beta+\mu)(\theta+\mu))\lambda+(\beta+\mu)(\theta+\mu)-R_0]=0 \tag{25}$$

According to Descartes' sign rule, equation (24) will have all negative roots if all the signs of each term are positive. Therefore, it is concluded that the disease-free equilibrium point is said to be stable if $R_0 \leq 1$ and unstable if $R_0 > 1$. ■

Theorem 2 *The endemic equilibrium point for the mathematical model of the Covid-19 spread is asymptotically stable.*

Proof.
The endemic equilibrium point applies to $I \neq 0$, and based on the J matrix in equation (22), a new matrix is acquired as seen in equation (26)

$$J = \begin{bmatrix} -(\alpha I^*(1-v)+\mu+v) & 0 & -\alpha S^*(1-v) & 0 \\ \alpha I^*(1-v) & -(\beta+\mu) & \alpha S^*(1-v) & 0 \\ 0 & \beta & -(\theta+\mu) & 0 \\ v & 0 & \theta & -\mu \end{bmatrix} \tag{26}$$

Afterward, the eigenvalues is acquired with the following description:

$$|\lambda I - J| = 0$$

$$|\lambda I - J| = \left| \left(\lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -(\alpha I^*(1-v)+\mu+v) & 0 & -\alpha S^*(1-v) & 0 \\ \alpha I^*(1-v) & -(\beta+\mu) & \alpha S^*(1-v) & 0 \\ 0 & \beta & -(\theta+\mu) & 0 \\ v & 0 & \theta & -\mu \end{bmatrix} \right) \right| = 0$$

$$|\lambda I - J| = \left| \begin{bmatrix} \lambda+(\alpha I^*(1-v)+\mu+v) & 0 & -\alpha S^*(1-v) & 0 \\ \alpha I^*(1-v) & \lambda+(\beta+\mu) & \alpha S^*(1-v) & 0 \\ 0 & \beta & \lambda+(\theta+\mu) & 0 \\ v & 0 & \theta & \lambda+\mu \end{bmatrix} \right| = 0$$

Thus, equation (27) is obtained as

$$0 = (\lambda+\mu)(\lambda+(\alpha I(1-v)+\mu+v))[(\lambda^2+(\beta+\mu)\lambda+(\theta+\mu)+(\beta+\mu)(\theta+\mu))-\alpha\beta S^*(1-v)+\alpha^2\beta SI(1-v)^2] \tag{27}$$

By substituting the endemic value of the equilibrium point, the following is attained:

$$0 = (\lambda+\mu)\left(\lambda+\frac{\alpha\beta\mu N}{(\beta+\mu)(\theta+\mu)}\right)(1-v)+\mu+v(\lambda^2+(\theta+\mu)+(\beta+\mu)\lambda)+(\alpha\beta\mu N-(\mu+v)(\theta+\mu)(\beta+\mu))(1-v) \tag{28}$$

According to Descartes' sign rule, if all the roots of the characteristic equation (λ) are positive, it can be concluded that the equilibrium point is Asymptotically Stable with the condition:

$$\alpha\beta\mu N > \mu+v \quad \text{and} \quad \frac{\alpha\beta\mu N}{\mu+v} > 1$$

5. Optimal Control Existence and Characteristics

The optimal control problem in this discussion relates to the steps of Covid-19 spread in South Sulawesi Province with the aim of reducing the number of individuals infected and increasing the number of individuals who recover. This vaccination is associated with the control variable u and added to the disease spread model with $u(t) \in U$ which is the percentage of individuals who have been vaccinated per unit time. Afterward, U is defined in terms of admissible control, which is defined as follows:

$$U = \{u(t) : 0 \leq u(t) \leq uend_{max}\} \tag{29}$$

The analysis used to prove the existence and characterization of vaccination control variables in cases of Covid-19 spread in South Sulawesi Province is the Pontryagin Principle with written evidence from step 1 to 5.

Step 1: Define the Objective Function (J)

Purpose function (J) = Minimize susceptible (S), infected (I) population and maximize cured (R) population

$$J(u) = \int_0^{\tau} S(t) + I(t) + \frac{1}{2}Cu^2(t)\, dt \tag{30}$$

with constraints/states in equation (1) -(4).

$C \geq 0$ is the weight coefficient to minimize the number of individuals susceptible and infected with Covid-19. The value of C is a counterweight to the control carried out, thus $u^*(t)$ is obtained,

$$J(u^*) = \min\{J(u); u \in U\} \tag{31}$$

with $U = \{u(t) : 0 \leq u(t) \leq 1, \forall t \in [0, T]\}$

Step 2: Forming the Hamiltonian Function

Hamiltonian function expressed in equation (32) is based on the objective function in (30) and constraints on equations (1) - (4).

$$\begin{aligned} H &= S(t) + I(t) + \frac{1}{2}Cu^2(t) + \sum_{i=1}^{4} \lambda_i(t) \cdot g_i(x(t), u(t), t) \\ H &= S(t) + I(t) + \frac{1}{2}Cu^2(t) + \lambda_1(t)(\mu N(t) - (\alpha I(t)(1-u) + \mu)S(t) - \mu(t-\tau) \cdot S(t-\tau)) \\ &\quad + \lambda_2(t)(\alpha I(t)S(t)(1-u) - (\beta+\mu)E(t)) + \lambda_3(t)(\beta E(t) - (\theta+\mu)I(t)) \\ &\quad + \lambda_4(t)(\theta I(t) + \mu(t-\tau) \cdot S(t-\tau) - \mu R(t)) \end{aligned} \tag{32}$$

Step 3: Finding the State and Coste Equations

According to Pontryagin's principle, the Hamiltonian function reaches an optimal solution if the state and costate equations as well as the stationary conditions are fulfilled.

1. State Equation

   This is attained by deriving the Hamiltonian function for each of its Lagrange multipliers (λ)

$$\dot{x_1} = \frac{\partial H}{\partial \lambda_1} = \mu N(t) - (\alpha I(t)(1-v) + \mu)S(t) - v \cdot S(t-\tau) \tag{33}$$

$$\dot{x_2} = \frac{\partial H}{\partial \lambda_2} = \alpha I(t)S(t)(1-v) - (\beta+\mu)E(t) \tag{34}$$

$$\dot{x_3} = \frac{\partial H}{\partial \lambda_3} = \beta E(t) - (\theta+\mu)I(t) \tag{35}$$

$$\dot{x_4} = \frac{\partial H}{\partial \lambda_4} = \theta I(t) + vS(s-\tau) - \mu R(t) \tag{36}$$

2. Costate Equation

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This is obtained by lowering the Hamiltonian function for each state variable/constraint (SEIR)

$$\dot{x_1} = -\frac{\partial H}{\partial S} = -1 - \lambda_1(t)[-\alpha I(t)(1-u) - \mu - v] - \lambda_2 \alpha I(t)(1-v) - \lambda_4 v \quad (37)$$

$$\dot{x_2} = -\frac{\partial H}{\partial E} = \lambda_2(\beta+\mu) - \lambda_3\beta \quad (38)$$

$$\dot{x_3} = -\frac{\partial H}{\partial I} = -1 + \lambda_1(t)\alpha(1-v)S(t) - \lambda_2 S(t)(1-v) + \lambda_3(\theta+\mu) + \lambda_4\theta \quad (39)$$

$$\dot{x_4} = -\frac{\partial H}{\partial R} = \lambda_4(t)\mu \quad (40)$$

With boundary conditions $\lambda_1(t) = \lambda_2(t) = \lambda_3(t) = \lambda_4(t) = 0$

Step 4: Determining the stationary conditions to get the optimal control form (u^*)

Stationary conditions are acquired by lowering the Hamiltonian function on the control variable $u(t)$.

$$\frac{\partial H}{\partial u} = 0$$

$$Cu(t) + \lambda_1(t)(\alpha I(t)S(t) - S(t-\tau)^2) - \lambda_2(t)\alpha I(t)S(t) + \lambda_4(t)S(t-\tau)^2 = 0$$

$$\vec{u}(t) = \frac{\lambda_1(t)(\alpha I(t)S(t) - S(t-\tau)^2) - \lambda_2(t)\alpha I(t)S(t) + \lambda_4(t)S(t-\tau)^2}{C}$$

$$\vec{u}(t) = \frac{\lambda_1\alpha IS(t) - \lambda_1 S(t-\tau)^2 - \lambda_2\alpha IS(t) + \lambda_4 S(t-\tau)^2}{C} \quad (41)$$

Because $0 \leq u(t) \leq 1$ then $u^*(t)$ is attained as

$$u^*(t) = \begin{cases} 0 & ,\vec{u} \leq 0 \\ \vec{u} & ,0 < \vec{u} < 1 \\ 1 & ,\vec{u} \geq 1 \end{cases}$$

Thus, the existence of optimal control is proven with the form $u^*(t)$ in order to optimize the objective function, as follows:

$$u^*(t) = \min\left\{\max\left\{0, \frac{\lambda_1\alpha IS(t) - \lambda_1 S(t-\tau)^2 - \lambda_2\alpha IS(t) + \lambda_4 S(t-\tau)^2}{C}\right\}\right\} \quad (42)$$

Step 5: Determining the Optimal State and Coste Equations

The optimal state and costate equations are established by substituting $u^*(t)$ in equation (42) to equation (33) -(40).

1. Optimal State Equation $x^*(t)$

$$x_1^*(t) = \frac{\partial H}{\partial \lambda_1}$$

$$\frac{\partial H}{\partial \lambda_1} = \mu N(t) - \left(\alpha I(t)\left(1 - \min\left\{\max\left\{0, \frac{\lambda_1\alpha IS(t) - \lambda_1 S(t-\tau)^2 - \lambda_2\alpha IS(t) + \lambda_4 S(t-\tau)^2}{C}\right\}, 1\right\}\right) + \mu\right) \quad (43)$$

$$x_2^*(t) = \frac{\partial H}{\partial \lambda_2}$$

$$\frac{\partial H}{\partial \lambda_2} = \alpha I(t)S(t)\left(1 - \min\left\{\max\left\{0, \frac{\lambda_1\alpha IS(t) - \lambda_1 S(t-\tau)^2 - \lambda_2\alpha IS(t) + \lambda_4 S(t-\tau)^2}{C}\right\}, 1\right\}\right)(\beta+\mu)E(t) \quad (44)$$

$$x_3^*(t) = \frac{\partial H}{\partial \lambda_3} = \beta E(t) - (\theta+\mu)I(t) \quad (45)$$

$$x_4^*(t) = \frac{\partial H}{\partial \lambda_4}$$

$$\frac{\partial H}{\partial \lambda_4} = \theta I(t) + \min\left\{\max\left\{0, \frac{\lambda_1\alpha IS(t) - \lambda_1 S(t-\tau)^2 - \lambda_2\alpha IS(t) + \lambda_4 S(t-\tau)^2}{C}\right\}, 1\right\}(t-\tau_1)\cdot S(t-\tau_2) - \mu R(t) \quad (46)$$

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2. Costate Equation

$$\begin{aligned}
\lambda_1^*(t) &= -\frac{\partial H}{\partial S} \\
&= -1 - \lambda_1(t)\begin{bmatrix} -\alpha I(t)\left(1 - \min\left\{\max\left\{0, \dfrac{\lambda_1\alpha IS(t) - \lambda_1 S(t-\tau)^2 - \lambda_2\alpha IS(t) + \lambda_4 S(t-\tau)^2}{C}\right\}, 1\right\}\right) \\ -\mu - \min\left\{\max\left\{0, \dfrac{\lambda_1\alpha IS(t) - \lambda_1 S(t-\tau)^2 - \lambda_2\alpha IS(t) + \lambda_4 S(t-\tau)^2}{C}\right\}, 1\right\}(t-\tau)^2 \end{bmatrix} \\
&\quad - \lambda_2\alpha I(t)\left(1 - \min\left\{\max\left\{0, \frac{\lambda_1\alpha IS(t) - \lambda_1 S(t-\tau)^2 - \lambda_2\alpha IS(t) + \lambda_4 S(t-\tau)^2}{C}\right\}, 1\right\}\right) \\
&\quad - \lambda_4 \min\left\{\max\left\{0, \frac{\lambda_1\alpha IS(t) - \lambda_1 S(t-\tau)^2 - \lambda_2\alpha IS(t) + \lambda_4 S(t-\tau)^2}{C}\right\}, 1\right\}(t-\tau)^2
\end{aligned} \tag{47}$$

$$\lambda_2^*(t) = -\frac{\partial H}{\partial E} = \lambda_2(\beta + \mu) - \lambda_3\beta \tag{48}$$

$$\begin{aligned}
\lambda_3^*(t) &= -\frac{\partial H}{\partial I} \\
&= -1 + \lambda_1(t)\alpha\left(1 - \min\left\{\max\left\{0, \frac{\lambda_1\alpha IS(t) - \lambda_1 S(t-\tau)^2 - \lambda_2\alpha IS(t) + \lambda_4 S(t-\tau)^2}{C}\right\}, 1\right\}\right) S(t) \\
&\quad - \lambda_2 S(t)\left(1 - \min\left\{\max\left\{0, \frac{\lambda_1\alpha IS(t) - \lambda_1 S(t-\tau)^2 - \lambda_2\alpha IS(t) + \lambda_4 S(t-\tau)^2}{C}\right\}, 1\right\}\right) + \lambda_3(\theta + \mu) - \lambda_4\theta
\end{aligned} \tag{49}$$

$$\lambda_4^*(t) = -\frac{\partial H}{\partial R} = \lambda_4(t)\mu \tag{50}$$

With boundary conditions $\lambda_1(t) = \lambda_2(t) = \lambda_3(t) = \lambda_4(t) = 0$

6. Simulation of the SEIR Mathematical Model for the Spread of Covid-19 in South Sulawesi

The SEIR Mathematical model simulation is carried out by using Maple software. Initial values $S(0), E(0), I(0), R(0)$ and parameter values of the model used are presented in Table 2 and 3.

Table 2. Initial Value of South Sulawesi Covid-19 SEIR Model

Variable	Value	Source
S	0.84618	(Ministry of Health Indonesia, 2018)
E	0.12979	(Ministry of Health Indonesia, 2018)
I	0.01214	(Ministry of Health Indonesia, 2018)
R	0.01189	(Ministry of Health Indonesia, 2018)

Table 3. Parameter values of the South Sulawesi Covid-19 SEIR Model

Variable	Value	Source
μ	0.00625	(Annas et al., 2020)
α	0.17	(Tanto & Handayani, 2022)
β	0.1005	(Annas et al., 2020)
θ	0.00150	(Ministry of Health Indonesia, 2018)
v	0.1; 0.9	(Ministry of Health Indonesia, 2018)

The equilibrium points of the SEIR model is determined by substituting the values of the uncontrolled parameters of Tables

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1 and 2 in Equation (1) - (4) which is equated to zero. Consequently, the following system of equations (51) - (54) is obtained as

$$\frac{dS}{dt} = 0.00625 - (0.17I + 0.00625)S \tag{51}$$

$$\frac{dE}{dt} = 0.17SI - 0.10675E \tag{52}$$

$$\frac{dI}{dt} = 0.1005E - 0.00775I \tag{53}$$

$$\frac{dR}{dt} = 0.00150I - 0.00625R \tag{54}$$

Equations (51) - (54) provide the equilibrium points for the SEIR model of the endemic COVID-19 spread, namely:

$$(S^*, E*, I^*, R^*) = (0.05380,\ 0.00499,\ 0.06479,\ 0.87640)$$

Furthermore, a simulation is carried out by adding a delay time to the SEIR mathematical model of the Covid-19 spread in South Sulawesi.

Simulation 1 ($\tau = 3$)

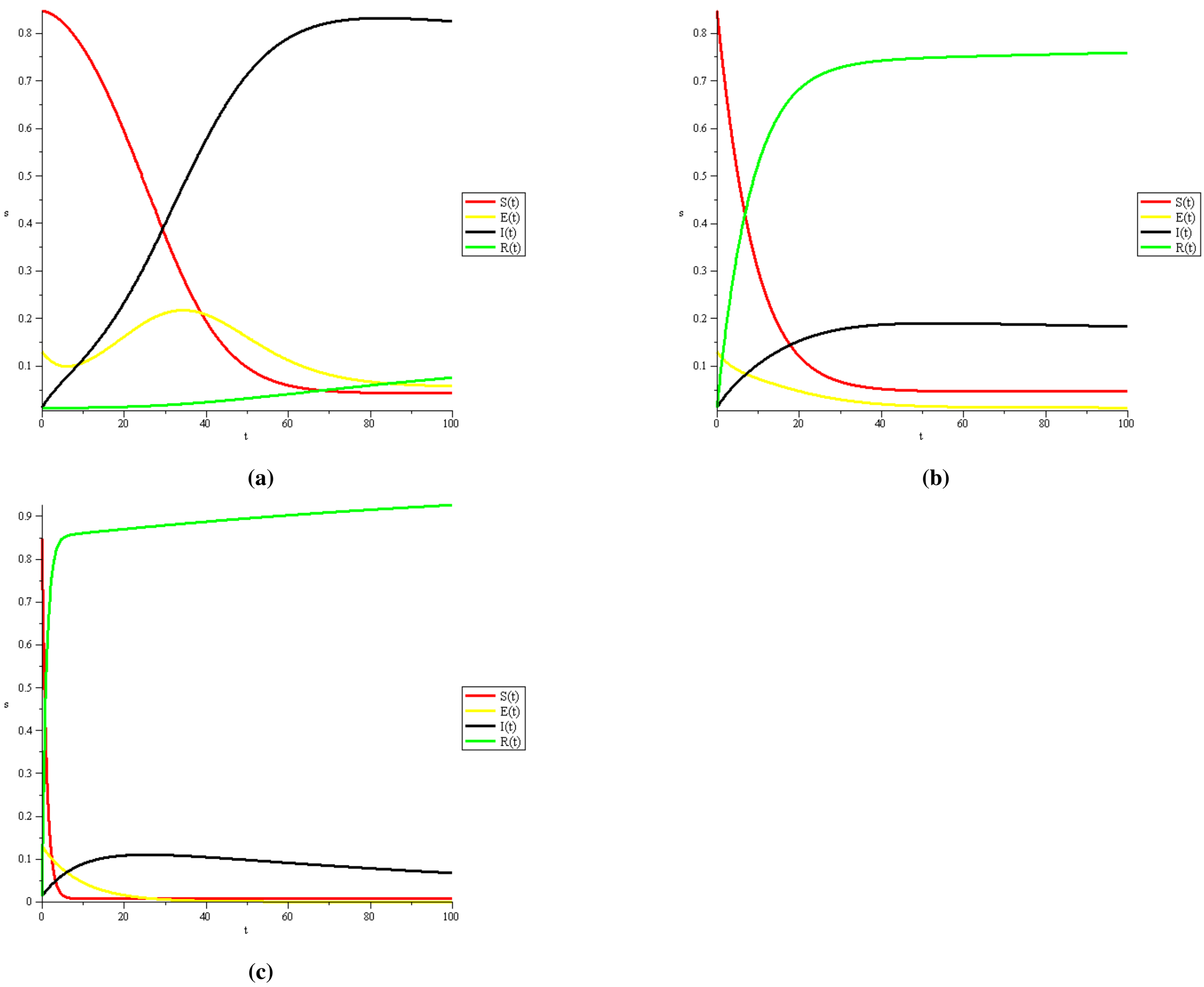
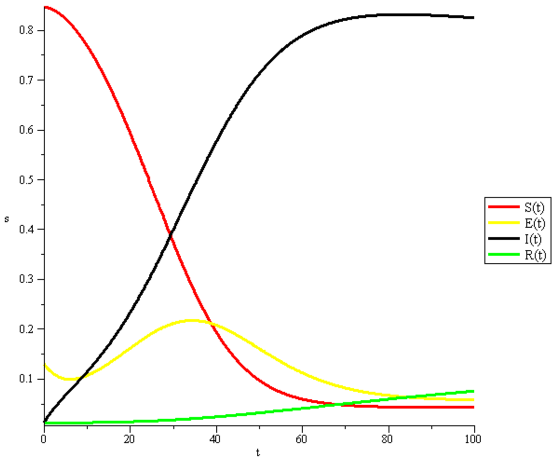


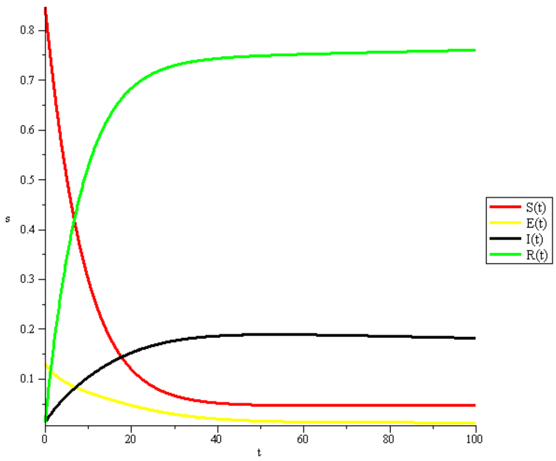
Figure 2. Simulation result graph $\tau = 3$ (a) without vaccine control (b) 10% vaccine (c) 90% vaccine

Based on simulations performed on 3 cases with a time delay of $\tau = 3$, the basic reproduction number value acquired include $R_0 = 20.689$ for cases without vaccination control, $R_0 = 1.0953$ for cases where vaccination was carried out on 10% of individuals in the population, and $R_0 = 0.0142$ for cases of vaccination carried out on 90% of the population.

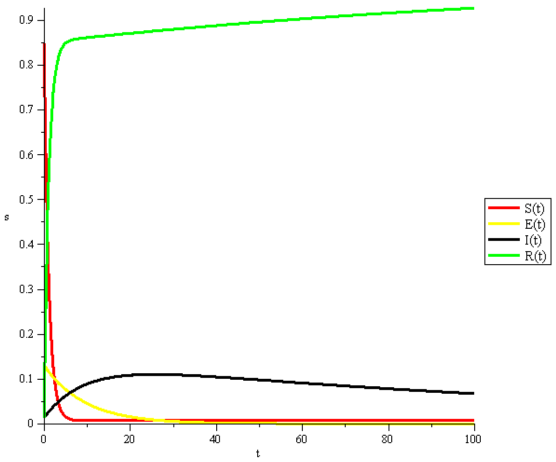
Simulation 2 ($\tau = 14$)



(a)



(b)



(c)

Figure 3. Simulation result graph $\tau = 14$ (a) without vaccine control (b) 10% vaccine (c) 90% vaccine

From the simulation carried out on 3 cases with a time delay of $\tau = 14$, the basic reproduction rate obtained, include $R_0 = 20.8322$ for cases without vaccination control, $R_0 = 1.1028$ for cases where vaccination is carried out on 10% of individuals, and $R_0 = 0.0143$ for cases of vaccination carried out on 90% of the population.

In general, the results of the SEIR mathematical model simulation of the Covid-19 spread in the South Sulawesi region is seen in Table 4.

Table 4. Simulation Results of the SEIR Model for the Spread of Covid-19

Case	τ	R_0
Without Vaccine Control	3	20.6899
	14	20.8322
Vaccine 10%	3	1.0953
	14	1.1028
Vaccine 90%	3	0.0142
	14	0.0143

Figure 2 and 3 showed that the number of transmissions that occur decreases with the increasing number of people who are vaccinated both for the time delay value of $\tau = 3$ and $\tau = 14$. Therefore, awareness is needed for the people to participate in the success of the government program, namely vaccination. This is conducted to control the transmission and further reduce the death rate.

D. DISCUSSION

The SIR and SEIR in (Annas et al., 2021) (Waziri et al., 2012) (Demirci et al., 2011) are used to build a Tuberculosis (TB) transmission model, which analyzes and predicts the number of TB cases in South Sulawesi. The dengue transmission model (DHF) employed in (Rangkuti et al., 2014) was used to analyze and predict dengue cases number, while that of Covid-19 carried out by (Annas et al., 2020) is the SEIR model in the territory of Indonesia. This SEIR model does not take into account the delay in analyzing the Covid-19 spread. Therefore, a model is built in this study to consider the delay time and optimal vaccination control in the population. The results of the simulation are very helpful to increase the strategic capabilities for controlling the number of Covid-19 cases because it provide information and forecast the number of cases.

E. CONCLUSION AND SUGGESTION

Based on the results, it can be concluded that the SEIR model with a time delay serve as a reference to see the spread of Covid-19 in South Sulawesi because the analysis provides evidence of the existence of optimal control. It is also seen that vaccination greatly affects the spread of virus, hence the people should be aware and follow the government's recommendation to get vaccinated in order to prevent or reduce the transmission rate of Covid-19.

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