Modified Hungarian Method for Solving Balanced Fuzzy Transportation Problems

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Article Info	ABSTRACT
	This paper discusses how to solve a balanced transportation problem, with transportation costs in the form of trapezoidal fuzzy numbers. Fuzzy costs are converted into crisp costs using Robust's method as a ranking function. A new modified approach of the Hungarian method has been applied to solve the balanced fuzzy transportation problem with the number of sources distinct from destinations. The analysis that we carry out comes from various literature studies and begins with examining the problem of fuzzy transportation, then collecting and connecting theories related to the problem of fuzzy transportation. The Hungarian method for the assignment problem was modified by adding some steps involving the principles of the transportation problem, such as that all that can be supplied will be supplied, in order to meet demand. This principle allows for one source to send products to multiple destinations and one destination can receive supplies from multiple sources. This is the basic concept of building this new method. This approach solves the fuzzy transportation problem in one optimization stage and produces the same results as other methods that solve the problem in two stages.

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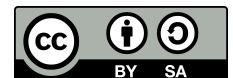
A. INTRODUCTION

The transportation problem is part of a wider class of linear problems, namely network flow. This problem can be solved using a transportation model based on the characteristics that a product is transported from a number of sources to a certain number of destinations with a minimum cost and optimized demand fulfilment. The basic model of the transportation problem assumes that each source can supply a number products and that each destination has a fixed amount demand (Taylor et al., 2013).

The Hungarian method proposed by (Kuhn, 1955) is one of the tools for solving a special type of transportation problem, namely the assignment problem. This method is more efficient in the iteration process than other methods. A disadvantage of this method is that it can only solve a balanced assignment problem, that is, the machines and the assignments have the same number. In order to solve the unbalanced assignment problem, it is necessary to add dummy machines which we will then ignore the work assigned to these machines. Given that the dummy is a pseudo activity, so the duration (cost) of the dummy activity (machines) is zero (Razi and Yudiarti, 2020). (Rabbani et al., 2019) proposed a different concept in solving the problem of unbalanced assignments, a modified Hungarian method which does not involve dummy workers (machines).

The Hungarian method uses deterministic data, which renders it unreliable for solving real problems that do not have definite and complete information. Fuzzy method becomes the best tool for solving problems with ambiguous information. This method is built based on the concept of fuzzy sets proposed by (Zadeh, 1965) and the concept of decision making involving fuzzy numbers (Bellman and Zadeh, 1970). Incorporating fuzzy numbers into the assignment problem provides a more realistic solution. (Kar et al., 2021) proposed a new approach which can solve fuzzy assignment problems using the Hungarian method.

Solving a more general fuzzy assignment problem, namely fuzzy transportation, has been carried out by several researchers such as (Patil and Chandgude, 2012), (Malini and Kennedy, 2013), and (Hunwisai and Kumam, 2017). These researchers solved the fuzzy transportation problem in two stages, namely determining a feasible solution and then ending with determining the optimal solution. This paper proposes a new approach to solve the fuzzy transportation problem in one optimization stage. This approach is built by modifying the Hungarian method.

B. LITERATURE REVIEW

1. Earlier Research

A modified method has been developed by (Kumar, 2006) to deal with unbalanced assignments. The unbalanced assignment cost matrix was split into two balanced parts and then solved using the Hungarian method. A similar approach was done by (Yadaiah et al., 2016) using the Lexi-search approach. (Betts et al., 2016) revised the numerical example provided by (Yadaiah et al., 2016) by retaining the original matrix of assignment costs and adding dummy rows to balance the assignments. The solution is carried out using the Hungarian method. (Younis and Alsharkasi, 2019) compare the use of the Hungarian method and the VAM method in solving transportation problems with the number of sources not the same as destinations. They added one dummy source so that it could be solved using the Hungarian method.

(Rabbani et al., 2019) modified the Hungarian method for solving unbalanced assignments without dummy variables. The results obtained are better than the modified Hungarian method applied by (Kumar, 2006), (Yadaiah et al., 2016), and (Betts et al., 2016) for the same problem. (Evipania et al., 2021) used a modified Hungarian method proposed by (Kumar, 2006) in solving unbalanced assignments for employees of Mitra Tex Convection.

The Hungarian Fuzzy approach in solving transportation problems with the same number of sources as the destinations was carried out by (Patil and Chandgude, 2012). The type of fuzzy numbers used was triangular fuzzy numbers (TFN) and the problem was solved using the MODI method. (Khalifa, 2020) solved transportation problems with heptagonal fuzzy numbers using the Goal Programming approach. In the same year (Srinivasan et al., 2020) solved the fuzzy transport problem for transporting material by utilizing a ranking function (beta distribution). (Manimaran and Ananthanarayanan, 2012) used Yager ranking on the fuzzy assignment problem using LINGO 9.0.

The solution to transportation problems with the number of sources distinct from the number of destinations was carried out by (Saman et al., 2020) using the Fuzzy Analytical Hierarchy Process (AHP) with triangular fuzzy numbers. (Dhanasekar et al., 2017) used the zero-point method and MODI with triangular fuzzy numbers, (Bisht and Srivastava, 2019) used One-Point Conventional, while (Aini et al., 2021) used the zero-suffix method with triangular fuzzy numbers.

2. Fuzzy Number

Definition 1. (Zimmermann, 1978) Let X be a set. A fuzzy set $\tilde{A}$ in X is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$, with $\mu_{\tilde{A}}(x)$ represents the degree of membership of x in $\tilde{A}$ in the interval $[0, 1]$.

Definition 2. (Bector et al., 2005) $\tilde{A}$ be a fuzzy set in R. Then $\tilde{A}$ is called a fuzzy number if:

1. $\tilde{A}$ is a convex, that is

$$\mu_{\tilde{A}}(\lambda\mu + (1 - \lambda)v) \geq \min\left(\mu_{\tilde{A}}(u), \mu_{\tilde{A}}(v)\right), \forall u, v \in R, \lambda \in [0, 1] \tag{1}$$

2. $\tilde{A}$ is normal
3. $\mu_{\tilde{A}}$ is upper semicontinuous, and
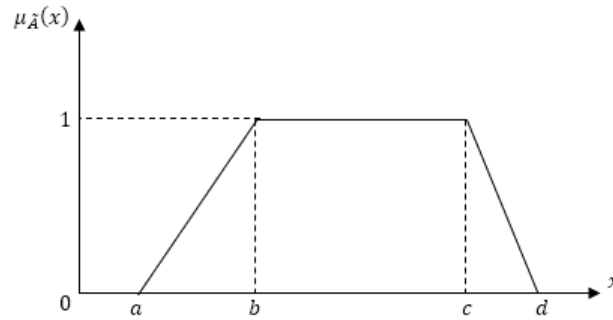4. $\tilde{A}$ has bounded support.

Figure 1. $TrFN\tilde{A} = (a, b, c, d)$

Definition 3. (Sakawa, 2013) Let α be a real number in $(0, 1)$ and $\tilde{A}$ be a fuzzy set. The α-level set of the fuzzy set $\tilde{A}$ is the set $\tilde{A}_\alpha = \{x \in X \ \mu_{\tilde{A}}(x) \geq \alpha\}$.

Definition 4. $TrFN\tilde{A} = (a, b, c, d)$ is a special fuzzy set in R, with membership function defined as the following (Figure 1):

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & , b \leq x \leq c \\ \dfrac{x - a}{b - a} & , a \leq x \leq b \\ \dfrac{d - x}{d - c} & , c \leq x \leq d \\ 0 & , \text{otherwise} \end{cases} \tag{2}$$

where $a \leq b \leq c \leq d$.

3. Robusts Ranking Technique

Definition 5. The Robusts ranking index for a convex fuzzy number $\tilde{A}$ is defined as

$$R\left(\tilde{A}\right) = \frac{1}{2} \int_0^1 \left[\tilde{A}_\lambda^L - \tilde{A}_\lambda^U\right] \, d\lambda \tag{3}$$

as for $TrFN \left[\tilde{A}_\lambda^L - \tilde{A}_\lambda^U\right] = (a + (b - a)\lambda) + (d - (d - c)\lambda)$

4. Balanced Fuzzy Transportation Problems

A fuzzy transportation problem with the number of sources distinct from the number of destinations is presented in Table 1.

Table 1. A fuzzy transportation problem with $m \neq n$

	D_1	D_2	$\ldots$	D_n	Supply
S_1	$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\ldots$	$\tilde{C}_{1n}$	SS_1
S_2	$\tilde{C}_{21}$	$\tilde{C}_{22}$	$\ldots$	$\tilde{C}_{2n}$	SS_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$\tilde{C}_{m1}$	$\tilde{C}_{m2}$	$\ldots$	$\tilde{C}_{mn}$	SS_2
Demand	SD_1	SD_2	$\ldots$	SD_n	$\sum\limits_{i=1}^{m} SS_i \sum\limits_{j=1}^{n} SD_j$

where

S_i	$=$	the ith source,
D_j	$=$	the jth destination,
$\tilde{C}_{ij}$	$=$	fuzzy transportation cost from source i to destination j,
SS_i	$=$	the maximum number of products can be transported from source i,
SD_j	$=$	demand from destination j.

A mathematical model for solving fuzzy transportation problem in Table 1 is

$$\text{Minimize } \tilde{Z} = \sum_i \sum_j \tilde{C}_{ij} X_{ij} \tag{4}$$

Subject to

$$\begin{aligned} \sum_j X_{ij} \leq SS_i, &\quad \forall i \\ \sum_i X_{ij} \geq SS_j, &\quad \forall j \\ X_{ij} \geq 0 &\qquad \forall i,j \end{aligned} \tag{5}$$

where X_{ij} is the number of products transported from source i to source j.

C. RESEARCH METHOD

The method used in this research is the analysis of theories relevant to fuzzy transportation problems with the number of sources not equal to the number of destinations. This analysis is sourced from various literature studies. This research begins by examining the problem of fuzzy transportation, then collecting and connecting theories related to the problem of fuzzy transportation. Furthermore, a new approach was developed to solve the fuzzy transportation problem in one optimization stage. The new approach in question is to modify the Hungarian method into a new method that can be applied to fuzzy transportation problems where the number of sources is not the same as the number of destinations. This new approach is then compared to the method used by (Hunwisai and Kumam, 2017). This new approach only uses one optimization stage, while (Hunwisai and Kumam, 2017)use two optimization stages, namely the initial basic feasible solution (IBFS) and the modified distribution method (MODIM).

D. RESULTS AND DISCUSSION

1. Algorithm

Given an unbalanced fuzzy transportation problem with m sources $S = \{S_1, S_2, \ldots, S_m\}$ and n destinations $D = \{D_1, D_2, \ldots, D_n\}$, where $m \neq n$. Let the fuzzy transportation cost from source i to destination j is $\tilde{C}_{ij}$. The rows of the table represent the m sources and the columns represents the n destinations.

Step-1 : Create an allocation table for the m sources and n destinations.

Step-2 : Insert the fuzzy transportation costs $\tilde{C}_{ij}$ into the allocation table according to their rows and columns.

Step-3 : Apply the Robust ranking technique to transform the fuzzy costs into crisp costs.

Step-4 : If there are more columns than rows, subtract the values in each column by the minimum cost in the column, then subtract the values in each row by the minimum cost in the row. Conversely, if there are more rows than columns, subtract the values in each row by the minimum cost in the row, then subtract each column by the minimum cost in the column. This Step-is to ensure that delivery is made on the lowest cost basis.

Step-5 : Test whether the ideal transportation has been achieved. Do this by determining that the minimum number of lines covering all zeros is equal to $\max\{m, n\}$ and that there is at least one intersecting lines (to ensure that at least one source can deliver the product to more than one destination or at least one destination can receive the product from multiple sources). If these criteria are met then proceed to Step-8, otherwise continue to step-6.

Step-6 : If the number of lines is less than $\max\{m, n\}$ or there are no intersecting lines, then choose the smallest cost that is not covered by a line then subtract each cost that is not covered by a line with the smallest cost that is not covered by a line. If any of the lines intersect at 0, add the smallest cost not covered by the 0 line.

Step-7 : Repeat Step-6 and 7 until the number of lines equals $\max\{m, n\}$ and there is at least one intersecting lines.

Step-8 : Test whether the supply in each row is no more than the demand in the columns containing zero for each row (to ensure that all products at each source can be delivered to the destination). If met, go to step-11, otherwise go to step-9.

Step-9 : If there is supply in a row that is less than the demand in the zero-loaded columns in that row, subtract each nonzero cost by the smallest nonzero cost.

Step-10 : Repeat Step-8 and 9 until the supply in each row is no less than the demand in the zero column for each of the same rows.

Step-11 : Delete all nonzero cells.

Step-12 : Find the row that contains only one zero, say row i, and find the column that contains that zero, say column j. Choose that zero and replace it with the number of products that can be delivered, i.e. $\min\{SS_i, SD_j\}$, then subtract SS_i and SD_j by the number of products sent (to find out the number of products that have not been delivered and the amount of demand that have not been fulfilled). If there is more than one row that contains only one zero, perform this Step-according to the largest supply. Do the same for columns that only have one zero. Delete the row if the supply in that row has been met and delete the column if the demand in that column has been met.

Step-13 : If Step-12 generates another row or column containing only one zero, repeat Step-12, if not, proceed to Step-14.

Step-14 : Select the row with the most supply (SS_i) that has not been transported and the column that has the most demand (SD_j) that has not been fulfilled. Put the value $\min\{SS_i, SD_j\}$ in the cell where supply and demand intersect. If there is more than one row that has the same remaining supply, select the row that has the least cost. If there is more than one column that has the same remaining demand, select the column containing the largest cost in row instead of i. Delete the row if the supply in that row has been met and delete the column if the demand in that column has been met.

Step-15 : Repeat Step-12 and 14 until every supply is fulfilled.

2. Numerical Example

1. Example 1

   In this example, we analyze a fuzzy transportation problem put forward by (Hunwisai and Kumam, 2017) using the algorithm explained earlier. This transportation problem involves four sources and three destinations. The transport costs are in the form of trapezoidal fuzzy numbers as presented in Table 2.

Table 2. The Fuzzy Transportation Problem from Example 1

Source	D_1	D_2	D_3	Supply (SS_i)
S_1	$(2,5,8,15)$	$(2,3,4,7)$	$(3,7,9,15)$	25
S_2	$(3,6,9,12)$	$(4,7,9,11)$	$(4,8,10,13)$	35
S_3	$(3,7,10,16)$	$(5,6,12,16)$	$(4,6,8,14)$	50
S_4	$(3,4,6,9)$	$(4,5,7,9)$	$(5,8,11,13)$	10
Demand (SD_j)	30	40	50	120

This is a balanced transportation problem since the quantity supplied equals that of the demand, namely 120.

Step-1 : Create the allocation table for $m = 4$ sources and $n = 4$ destinations (Table 2).

Step-2 : Fill the $\tilde{C}_{ij}$ values in the table accordingly (Table 2).

Step-3 : Apply the Robust ranking technique to transform the fuzzy costs into crisp costs.

The fuzzy costs to transport a product from S_1 to D_1 in Table 2 are $(2, 5, 8, 15)$. Since $[\tilde{C}_{11\lambda}^L - \tilde{C}_{11\lambda}^U] = (2+3\lambda) + (15-7\lambda) = 17 - 4\lambda$, we have

$$R\left(\tilde{C}\right) = (2,5,8,15) = \frac{1}{2}\int_0^1 [\tilde{C}_{11\lambda}^L - \tilde{C}_{11\lambda}^U]\, d\lambda = \frac{1}{2}\int_0^1 (17 - 4\lambda)\, d\lambda = 7.5 \tag{6}$$

The remaining $\tilde{C}_{ij}$ values are calculated in similar fashion, hence we obtain $R(\tilde{C}_{12}) = 4$, $R(\tilde{C}_{13}) = 8.5$, $R(\tilde{C}_{21}) = 7.5$, $R(\tilde{C}_{22}) = 7.75$, $R(\tilde{C}_{23}) = 8.75$, $R(\tilde{C}_{31}) = 9$, $R(\tilde{C}_{32}) = 9.75$, $R(\tilde{C}_{33}) = 8$, $R(\tilde{C}_{41}) = 5.5$, $R(\tilde{C}_{42}) = 6.25$, and $R(\tilde{C}_{43}) = 9.25$. This gives us Table 3:

Table 3. Fuzzy Transportation After Ranking

Source	D_1	D_2	D_3	Supply (SS_i)
S_1	7.5	4	8.5	25
S_2	7.5	7.75	8.75	35
S_3	9	9.75	8	50
S_4	5.5	6.25	9.25	10
Demand (SD_j)	30	40	50	120

Step-4 : Since there are more rows than columns, we subtract each row by the minimum cost in the row, thereafter subtract each column by minimum cost in the column (Table 4).

Table 4. Step-4

Source	D_1	D_2	D_3	Supply (SS_i)
S_1	2.5	0	3.25	25
S_2	0	0	0	35
S_3	0	1.5	0	50
S_4	0	0.5	2.5	10
Demand (SD_j)	30	40	50	120

Step-5 : An ideal transport has been achieved since the number of minimum lines covering all zeros is four, the same as $\max\{m,n\}$. There are also intersecting lines (Table 5).

Table 5. Step-5

Source	D_1	D_2	D_3	Supply (SS_i)
S_1	2.5	0	3.25	25
S_2	0	0	0	35
S_3	0	1.5	0	50
S_4	0	0.5	2.5	10
Demand (SD_j)	30	40	50	120

We then proceed to **Step-8**, as suggested by the algorithm. From Table 5, supplies in each row are no more than demands in columns containing zeros for each row. This implies all products from each source can be delivered to the destinations and hence we can go to **Step-11**.

Step-11 : Delete all nonzero cells (Table 6).

Table 6. Step-11

Source	D_1	D_2	D_3	Supply (SS_i)
S_1		0		25
S_2	0	0	0	35
S_3	0		0	50
S_4	0			10
Demand (SD_j)	30	40	50	120

Step-12 : We can see there are two rows that have one zero, namely S_1 and S_4 with SS_1 having the biggest demand, namely 25. The demand that contains 0 is SD_2, namely 40. Replace that 0 with $\min\{SS_1, SD_2\} = 25$ and repeat this on row S_4. We see that SS_1 become $25 - 25 = 0$ and SD_1 become $40 - 25 = 15$ respectively (Table 7).

Table 7. Step-12

Source	D_1	D_2	D_3	Supply (SS_i)
S_1		25		0
S_2	0	0	0	35
S_3	0		0	50
S_4	10			0
Demand (SD_j)	20	15	50	85

Step-13 : Step-12 leaves column D_2 with only one zero. Using the same treatment as in Step-12 we obtain Table 8.

Table 8. Step-14

Source	D_1	D_2	D_3	Supply (SS_i)
S_1		25		0
S_2	0	15	0	20
S_3	0		0	50
S_4	10			0
Demand (SD_j)	20	0	50	70

Step-14 : Notice that row S_2 and S_3 have more than one zero. Choose S_3 since it has the most demand, namely $SS_3 = 50$.

SS_3 can deliver a product to D_1 and D_3. Choose D_3 since it has the most demand, namely $SD_3 = 50$. Replace the value in the cell of intersection between S_3 and D_3 with $\min\{50, 50\} = 50$ then subtract SS_3 and D_3 by that value (Table 9).

Table 9. Step-14

Source	D_1	D_2	D_3	Supply (SS_i)
S_1		25		0
S_2	0	15		20
S_3			50	0
S_4	10			0
Demand (SD_j)	20	0	0	20

Step-15: Only one zero left, namely in the cell of intersection between S_2 and D_1. Replace that 0 with $\min\{SS_2, SD_1\} = 20$ and subtract SS_2 and SD_1 by 20 (Table 10). We see that SS_i and SD_j becomes zero, which means supplies and demand have been met.

Table 10. Step-15

Source	D_1	D_2	D_3	Supply (SS_i)
S_1		25		0
S_2	20	15		0
S_3			50	0
S_4	10			0
Demand (SD_j)	0	0	0	0

Table 11 shows the best transport decision to deliver all products from every source and meets demand from every destination.

Table 11. The Best Transport Decision

Source	D_1	D_2	D_3	Supply (SS_i)
S_1		4(25)		100
S_2	7.5(20)	7.75(15)		266.25
S_3			8(50)	400
S_4	5.5(10)			55
Demand (SD_j)	205	216.25	400	825.25

The optimal fuzzy transport cost is $4(25) + 7.5(20) + 7.75(15) + 8(50) + 5.5(10) = 821.25$, which agrees with that which obtained by (Hunwisai and Kumam, 2017).

2. Example 2

This time we take a transportation problem put forward by (Balasubramanian and Subramanian, 2018), where there are more destinations than sources. The transport costs, supply and demand are in the form of trapezoidal fuzzy number and presented in Table 12.

Table 12. Fuzzy Transportation Problem

Source	D_1	D_2	D_3	D_4	Supply (SS_i)
S_1	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
S_2	(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
S_3	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
Demand (SD_j)	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Results obtained from Robusts ranking technique (Step-3) can be found in Table 13.

Table 13. Step-3

Source	D_1	D_2	D_3	D_4	Supply (SS_i)
S_1	2.5	3.5	11.5	7.5	6.5
S_2	1.75	0.5	6.5	1.5	1.5
S_3	5.5	8.5	15.5	9.5	11
Demand (SD_j)	7.5	5.5	3.5	2.5	19

Step-4 : Since the number of columns is more than the number of rows, subtract each column by the minimum cost in the column, then subtract each row by the minimum cost in the row (Table 14).

Table 14. Step-4

Source	D_1	D_2	D_3	D_4	Supply (SS_i)
S_1	0	2.25	4.25	5.25	6.5
S_2	0	0	0	0	1.5
S_3	0	4.25	5.25	4.25	11
Demand (SD_j)	7.5	5.5	3.5	2.5	19

This transportation is not ideal, so we revert to Step-6 and 7, and obtain Table 15.

Table 15. Step-6 and 7

Source	D_1	D_2	D_3	D_4	Supply (SS_i)
S_1	0	0	0	1	6.5
S_2	2.25	2	0	0	1.5
S_3	0	2	1	0	11
Demand (SD_j)	7.5	5.5	3.5	2.5	19

Step-8: Row SS_3 is bigger than $SD_1 + SD_3$, hence we proceed to Step-9 and obtain Table 16.

Table 16. Step-8 and 9

Source	D_1	D_2	D_3	D_4	Supply (SS_i)
S_1	0	0	0	1	6.5
S_2	2.25	2	0	0	1.5
S_3	0	1	0	0	11
Demand (SD_j)	7.5	5.5	3.5	2.5	19

Continuing to Step-11, we delete all nonzero cells and do Step-12, which gives Table 17.

Table 17. Step-11 and 12

Source	D_1	D_2	D_3	D_4	Supply (SS_i)
S_1	0	5.5	0		1
S_2			0	0	1.5
S_3	0		0	0	11
Demand (SD_j)	7.5	0	3.5	2.5	13.5

Proceed to Step-14 and we get Table 18.

Table 18. Step-14

Source	D_1	D_2	D_3	D_4	Supply (SS_i)
S_1		5.5	0		1
S_2			0	0	1.5
S_3	7.5		0	0	3.5
Demand (SD_j)	0	0	3.5	2.5	6

Since row S_1 contains only one zero, we perform Step-12 for this row and get Table 19.

Table 19. Step-12

Source	D_1	D_2	D_3	D_4	Supply (SS_i)
S_1		5.5	1		0
S_2			0	0	1.5
S_3	7.5		0	0	3.5
Demand (SD_j)	0	0	2.5	2.5	5

We continue to Step-14. Since S_3 has the most supply, it becomes a priority. Since $SD_3 = SD_4$ we choose $D_4(9.5)$ since it has the least transport cost in S_3 (see Table 13). We then obtain Table 20.

Table 20. Step-14 again

Source	D_1	D_2	D_3	D_4	Supply (SS_i)
S_1		5.5	1		0
S_2			0		1.5
S_3	7.5		0	2.5	1
Demand (SD_j)	0	0	2.5	0	2.5

Again, we perform Step-12 and obtain the optimal transportation table (Table 21).

Table 21. Step-12 again

Source	D_1	D_2	D_3	D_4	Supply (SS_i)
S_1		5.5	1		0
S_2			1.5		0
S_3	7.5		1	2.5	0
Demand (SD_j)	0	0	0	0	0

The best transportation decision is presented in Table 22.

Table 22. The Best Transportation Decision

Source	D_1	D_2	D_3	D_4	Supply (SS_i)
S_1		3.5(5.5)	11.5(1)		30.75
S_2			6.5(1.5)		9.75
S_3	5.5(7.5)		15.5(1)	9.5(2.5)	80.5
Demand (SD_j)	41.25	19.25	36.75	23.75	121

The optimal fuzzy transportation cost is 121, which agrees with that which obtained by (Balasubramanian and Subramanian, 2018).

E.   CONCLUSION AND SUGGESTION

The modified Hungarian method can solve the fuzzy transportation problem with the number of sources not equal to the number of destinations. The optimal solution is obtained in one step. This approach yields the same results as other methods that solve the problem in two stages. Future studies can develop this approach to solve fuzzy transportation problems without transforming fuzzy costs into crisp costs. In addition, it is necessary to develop methods for fuzzy transportation probslems involving fuzzy supply and fuzzy demand.

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