Expansion of Stock Portfolio Risk Analysis Using Hybrid Monte Carlo-Expected Tail Loss

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ABSTRACT

Monte Carlo-Expected Tail Loss (MC-ETL) is the new expansion method that combines simulation and calculation to measure investment risk. This study models US stock prices using ARIMA-GARCH and forms an optimized portfolio based on Multi-Objective that aims to analyze the portfolio investment return. The next portfolio return will be simulated using the Monte Carlo (MC) method, measured based on the Expected Tail Loss (ETL) calculation. The optimized portfolio comprises 5 US stocks from 10 years of data, with the biggest capitalization market on February 25, 2021. MSFT has the most considerable weight in the optimized portfolio, followed by GOOG, AAPL, and AMZN, whereas TSLA shares have negligible weight. Based on the simulation result, the optimized portfolio has the smallest ETL value compared to its constituent stocks, which is ±0.029 or about 2.9%. This value means that the optimized portfolio is concluded as an investment choice for investors with a low level of risk.

A. INTRODUCTION

The state of a country’s economy will be directly proportional to the state of the capital market. A capital market is a meeting place for investors as parties who have excess funds and issuers as parties who need funds (Wardiyah, 2017). At the end of 2020, International Financial Statistics stated that the United States was one of the largest and most stable economies. Thus, these conditions significantly affected the price stability of financial instruments in the United States capital market. One of the financial instruments in the United States capital market that has price stability is the type of stock. In addition, stocks are the most sought-after financial instrument by investors (Sururi et al., 2021). That is most likely why many investors are interested in investing in US stocks.

Investment activities certainly cannot be separated from risks. When viewed from the level of risk, investment in the capital market is known to have a higher level than other instruments such as deposits, bonds, mutual funds, and others (Bellofatto et al., 2018). In investing, investors aim to get high returns with low risk (Mayuni and Suarjaya, 2018). In actual practice, this objective contradicts the relationship between return and risk, which states that the higher the expected return, the higher the risk to be faced. Therefore, it is necessary to invest in a model that can describe the volatility of returns.

The return volatility of a stock will explain the risk of a given stock return. Research related to investment volatility modeling has been conducted by (Azmi and Syaifudin, 2020). This study applies the ARIMA-GARCH model to estimate commodity prices, concluding that the model obtained has an error rate of less than 2% for all types of commodities used (Azmi and Syaifudin, 2020). The process of minimizing the volatility of stock returns can be done by forming an investment portfolio (Salim and Rizal, 2021).
Stock Portfolio Optimization Approach using Multi-Objective Optimization has been proposed by Chen et al. (2018). The study uses a real dataset to test the effectiveness of the proposed approach (Zheng and Chen, 2013).

The Expected Tail Loss (ETL) method was used to estimate risk on the Malaysian market index by Nguyen and Huynh (Nguyen and Huynh, 2019), with the result that the ETL risk value was 2.26%. Then a similar study using ETL to compare cryptocurrency risk by Feng et al. (Feng et al., 2018) resulted in 7.92%, 14.12%, and 1.86% ETL values for Bitcoin, Ethereum, and S&P500, respectively. Conditional Value at Risk (CVaR), another name for ETL, is used by Arif and Sohail (Arif and Sohail, 2020) to estimate the risk of the Pakistan stock exchange with an optimal investment ETL value of 7.52%. Carried out the ETL method during the covid-19 pandemic on Nordic stock markets, which stated that the ETL value was over 20% on OMXS30 data and over 15% on OMXH25 and OMXS30 during the Covid-19 crisis.

Even though the investment portfolio has been formed, the risk of loss will inevitably remain. This risk can be predicted by obtaining the characteristics and distribution of investment returns. This study has an update that adds a simulation process using Monte Carlo in optimized portfolio returns to get a stable return. After iteration is done, Expected Tail Loss (ETL) is used in the optimized portfolio return to measure investment risk.

B. LITERATURE REVIEW

1. US Big-Cap Stocks

A stock market is a meeting place for sellers and buyers involved in stock economic transactions. The stock market is also an integral and inseparable part of a country’s economy. The stock market is also an integral and inseparable part of a country’s economy. The United States is one of the countries with the largest and most developed financial market, one of the instruments is stocks (Zheng and Chen, 2013).

There are three major stock market indexes in the United States: the DJIA, S&P 500, and NASDAQ. The DJIA is one of the most important economic indices globally, with 30 of the largest blue-chip multinational companies’ stocks included in its components. However, the DJIA index is narrower or less comprehensive in scope than the S&P 500 index (Jareño et al., 2016). The S&P 500 index uses a market capitalization methodology. It has covered 500 companies in large-cap market sectors, so the S&P 500 index is usually used as a benchmark for the performance of large-cap stocks and the influence of US stock prices on other countries.

On the other hand, the NASDAQ stock index includes many small, high-growth stocks. This makes the NASDAQ stock more volatile when compared to the other two stock indices (Zheng and Chen, 2013). Companies that are members of the NASDAQ stock index are mostly in the technology sector. The NASDAQ stock index is compiled based on the capitalization method, which determines the weight of each stock. From this explanation, the stock index with large capitalization is the NASDAQ stock index.

2. Portfolio

A portfolio is a term for a combination of some securities. Portfolios mainly deal with the problem of how to allocate one’s capital to a large number of securities so that investments can bring about the most profitable returns (Chandra, 2017). Portfolio analysis is a quantitative method for selecting the optimized portfolio to maximize returns and minimize risk in various uncertain environments. To choose the optimized portfolio, we must first answer the questions “what is the portfolio’s rate of return” and “what is the risk of the portfolio”. Not only about how large the capital is, but preliminary evidence to suggest that the portfolio’s overall performance may be favorable because the experienced low correlation of the impact of investments to traditional markets reduces portfolio risk and increases sustainability (Brandstetter and Lehner, 2015).

Let the asset weight vector \( \vec{x} = [x_1, x_2, \ldots, x_n]^T \) with \( x_i \) as the weight of asset \( i \) in the portfolio and the expected return for each asset in the portfolio is expressed in the vector form \( \vec{r} = [r_1, r_2, \ldots, r_n]^T \) with \( r_i \) as the mean return of asset \( i \) in the portfolio. The portfolio’s expected return is the weighted average of the returns on individual assets is expressed as

\[
x_p = \vec{r}^T \vec{x} = \sum_{i=1}^{n} x_i r_i
\]

The variance and covariance of individual assets are characterized by the variance-covariance matrix \( \Sigma = \)
where \( n \) is important. For a multi-asset portfolio, the number of assets equals \( n \) may be responsible for many investments. Therefore, extending the analysis to the return of portfolios with more than two assets is important. For a multi-asset portfolio, the number of assets equals \( n \), and \( x_i \) represents the proportion of funds invested in each. The portfolio return is as follows

\[
R_p = \sum_{i=1}^{n} x_i R_i
\]

where \( R_i \) is the \( i \)-th asset return and \( R_p \) is the portfolio return (Kulali, 2016).

3. ARIMA-GARCH Model

Let \( \{z_t\} \) represent the observed time series observation data at spaced times \( t \), also let \( \tilde{z}_t = z_t - \mu \) be the series of deviations from \( \mu \). In addition, let \( \{a_t\} \) represent the unobserved white noise series, i.e., a sequence of identically distributed independent random variables with zero mean. In most studies, the assumption of independence can be replaced by a weaker assumption that \( \{a_t\} \) is an uncorrelated random variable. The Mixed Autoregressive Moving Average Model (ARMA) assumes that the time series observation data is partly autoregressive and partly moving average. In general, the ARMA \((p, q)\) model is stated as follows.

\[
\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \cdots + \phi_p \tilde{z}_{t-p} + a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q}
\]

or

\[
\theta(B) \tilde{z}_t = \phi(B) a_t
\]

where \( B \) is the backward shift operator such that \( \phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p \) and \( \theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q \), \( \phi \) and \( \theta \) are Autoregressive (AR) and Moving Average (MA) parameter. The model uses \( p + q + 2 \) unknown parameters such as \( \mu, \phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q, \sigma_a^2 \), that are estimated from the observation data (Box et al., 2015).

Many series encountered in an industry or business (e.g., stock prices) exhibit nonstationary behavior and typically do not vary about a fixed average. The process provides a robust model for describing stationary and nonstationary time series and is called an Autoregressive Integrated Moving Average Process, order \((p, d, q)\), or ARIMA\((p, d, q)\) process. The process can be written as follows

\[
w_t = \phi_1 w_{t-1} + \phi_2 w_{t-2} + \cdots + \phi_p w_{t-p} + a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q}
\]

where \( w_t = (1 - B)^d z_t \) and note to change \( w_t \) to \( z_t - \mu \) when \( d = 0 \) (Box et al., 2015).

For a stationary ARIMA process, the unconditional mean of the series is constant over time, while the conditional mean of \( E[z_t | F_{t-1}] \) varies as a function of the previous observations. Parallel to this, the ARCH model assumes that the unconditional variance of the error process is constant over time but allows the conditional variance of \( a_t \) to vary as a function of the past squared error. Let \( \sigma_t^2 = Var(a_t | F_{t-1}) \) denote the conditional variance, given the last \( F_{t-1} \), the basic model ARCH(s) can be formulated as

\[
a_t = \sigma_t e_t
\]

where \( \{e_t\} \) is a sequence of iid random variables with mean zero and variance one. So that

\[
\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_s a_{t-s}^2
\]

where \( \alpha \) is ARCH parameter with \( \alpha_0 > 0, \alpha_i \geq 0 \) for \( i = 1, 2, \ldots, s - 1 \), and \( \alpha_s > 0 \) (Box et al., 2015).
The ARCH model has the disadvantage of often requiring a sequence of high lags to describe the evolution of volatility over time adequately. An extension of the ARCH model is called generalized ARCH or GARCH. The GARCH \((s, r)\) is given by

\[
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{s} \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^{r} \beta_j \sigma_{t-j}^2
\]  

(9)

where \(\alpha\) and \(\beta\) are GARCH parameter with \(\alpha_0 > 0, \alpha_i \geq 0\) for \(i = 1, 2, \ldots, s\), \(\alpha_s \geq 0\), \(\beta_j \geq 0\) for \(j = 1, 2, \ldots, r\), and \(\beta_r > 0\) (Box et al., 2015).

4. Multi-Objective Optimization

The concept of multi-objective optimization was first introduced by the French-Italian economist Pareto (Lampo, 2018). This theory combines all objectives into one objective function, and the standard solution method for minimizing the total objective is applied as follows.

\[
\begin{align*}
\min F(x) &= [f_1(x), f_2(x), \ldots, f_m(x)] \\
\text{subject to:} & \\
G(x) &= [g_1(x), g_2(x), \ldots, g_k(x)] < 0 \\
H(x) &= [h_1(x), h_2(x), \ldots, h_l(x)] = 0
\end{align*}
\]

(10) (11) (12)

where \(F(x)\) is a vector of \(m\) objective functions with \(f_i(x)\) is the objective function \(i\) for \(i = 1, 2, \ldots, m\), \(G(x)\) is a vector of \(k\) inequality constraints with \(g_i(x)\) is the inequality constraint \(i\) for \(i = 1, 2, \ldots, k\), \(H(x)\) is a vector of \(l\) equality constraints with \(h_i(x)\) is the equality constraint \(i\) for \(i = 1, 2, \ldots, l\), and \(x\) is a vector of decision variables with \(x = [x_1, x_2, \ldots, x_n]\) (Liang et al., 2016).

5. Monte Carlo Simulation

Suppose the simulation in this study aims to estimate an unknown quantity \(l\) based on the \(\hat{l}\) estimator, which is a function of the data generated by the simulation. The general condition is when \(l\) is the expectation of the output variable \(Y\) from the simulation. For example, a simulation experiment that is run repeatedly produces independent copies of \(Y_1, \ldots, Y_N\) of \(Y\). A statistical estimator of \(l\) then the sample mean is as follows

\[
\hat{l} = \bar{Y} = \frac{1}{N} \sum_{i=1}^{N} Y_i
\]

(13)

This estimator is unbiased in the sense that \(E[\hat{l}] = l\). Moreover, according to the law of large numbers, \(\hat{l}\) converges to \(l\) with probability one as \(N \to \infty\). Note that the estimator is viewed as a random variable. Certain results or observations are called estimates (numbers), often denoted by the same letter (Ling and Rubinstein, 1983).

6. Expected Tail Loss

The Expected Tail Loss (ETL), also called Conditional Value at Risk (CVaR), interprets the expected loss (in present value terms) given that the loss exceeds the Value at Risk (VaR). VaR indicates the maximum loss of an asset or portfolio of assets in a specific confidence interval (Siswono et al., 2021). The ETL risk metric is more informative than VaR because VaR does not measure the extent of extraordinary losses. VaR states the level of loss that we believe will not be exceeded: it does not tell us how much could be lost if VaR is exceeded. However, the ETL tells us how much loss we can expect, given that the VaR is exceeded. ETL provides a complete picture of portfolio risk than simply reporting VaR alone. This means that ETL is a better risk metric for regulatory and economic capital allocation. Suppose the distribution function \(F_X\) for a certain probability, the VaR value of \(X\) denoted \(VaR(F_X, \alpha)\) can be calculated as follows:

\[
VaR(F_X, \alpha) = VaR_{\alpha}(X) = -F_X^{-1}(1 - \alpha)
\]

(14)
Thus, the ETL value, which is expressed as a measure of portfolio risk, can be written as follows:

$$ETL_\alpha = -E[X | X < -VaR_\alpha(X)]$$

(15)

$$ETL_\alpha = -\int_{-\infty}^{-VaR} xF_X(x) \, dx$$

(16)

where $\alpha$ is the quantil of the distribution of $X$ (Airouss et al., 2018).

### C. RESEARCH METHOD

This study takes five US stocks that have the largest market capitalization on February 25, 2021, namely Apple (AAPL), Microsoft (MSFT), Alphabet/Google (GOOG), Amazon (AMZN), and Tesla (TSLA). The research was conducted using ten years of data from January 2, 2012, to December 31, 2021, obtained through finance.yahoo.com. The entire analysis was carried out using R software, especially the PerformanceAnalytics packages, to perform Monte Carlo simulation and calculate the Expected Tail Loss (MC-ETL).

The steps in the research are as follows:

1. Analyze descriptive statistics to see the distribution of each stock data.
2. Modeling the daily closing price of each stock using ARIMA-GARCH.
3. Calculating the mean-return of each stock based on the ARIMA-GARCH model.
4. Forming an optimized portfolio using the Multi-Objective Optimization Method.
5. Perform a Monte Carlo simulation on optimized portfolio returns.
6. Calculate the ETL value of the optimized portfolio for each simulation.
7. Calculate the mean value of ETL from all simulations that have been carried out.
8. Calculating the individual ETL value of each portfolio stock.
9. Comparing the optimal ETL portfolio value with the individual ETL of each constituent stock.
10. Interpret the results obtained and draw conclusions.

### D. RESULTS AND DISCUSSION

1. Data Characteristics

Data used daily closing price data for five US stocks with the largest market capitalization. The data characteristics of the data, as a sample, are presented in descriptive statistics as follows, presented in Table 1.

<table>
<thead>
<tr>
<th>Stock</th>
<th>AAPL (USD)</th>
<th>MSFT (USD)</th>
<th>GOOG (USD)</th>
<th>AMZN (USD)</th>
<th>TSLA (USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Cap</td>
<td>2.69T</td>
<td>2.229T</td>
<td>1.778T</td>
<td>1.565T</td>
<td>837B</td>
</tr>
<tr>
<td>Sample</td>
<td>2516</td>
<td>2516</td>
<td>2516</td>
<td>2516</td>
<td>2516</td>
</tr>
<tr>
<td>Mean</td>
<td>48.99</td>
<td>97.92</td>
<td>988.1</td>
<td>1245.7</td>
<td>140.941</td>
</tr>
<tr>
<td>Median</td>
<td>32.36</td>
<td>62.69</td>
<td>798.2</td>
<td>818.7</td>
<td>49.783</td>
</tr>
<tr>
<td>Minimum</td>
<td>13.95</td>
<td>26.37</td>
<td>278.5</td>
<td>175.9</td>
<td>4.558</td>
</tr>
<tr>
<td>Maximum</td>
<td>180.33</td>
<td>343.11</td>
<td>3014.2</td>
<td>3731.4</td>
<td>1229.91</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>38.65549</td>
<td>78.15647</td>
<td>628.0657</td>
<td>1052.281</td>
<td>237.3425</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.567776</td>
<td>1.340117</td>
<td>1.476066</td>
<td>0.9001429</td>
<td>2.417873</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.329771</td>
<td>3.855755</td>
<td>4.870672</td>
<td>2.538728</td>
<td>7.884726</td>
</tr>
</tbody>
</table>

From the table above, it can be seen that each stock has different statistical values or characteristics. The difference in value can lead to differences in the return and risk of each stock. As an illustration, the price movement of each stock is shown in Figure 1.
The existence of a downward movement or graph of stock prices in Figure 1 defines that the value of a stock is experiencing a decline. These events are events that investors must avoid. Therefore, we will try to measure how much impact the price decline has on the investment value of an investor, of course, based on all price movements and volatility.

2. ARIMA-GARCH Estimation

Before doing the modeling, the first step that must be done is to test the data to obtain stationarity information from the data. The test was carried out using the Augmented Dickey-Fuller Unit Root Test. If the data is stationary, then the modeling can be continued immediately, but if the data is not stationary, a differencing process is needed to make the data stationary. With the null hypothesis that the data is not stationary, the results of the stationarity test are shown in Table 2.
Table 2. Stationary Test of Each Data

<table>
<thead>
<tr>
<th>Stock</th>
<th>t-Statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>3.022902</td>
<td>1.0000</td>
</tr>
<tr>
<td>MSFT</td>
<td>3.433418</td>
<td>1.0000</td>
</tr>
<tr>
<td>GOOG</td>
<td>2.385097</td>
<td>1.0000</td>
</tr>
<tr>
<td>AMZN</td>
<td>0.345716</td>
<td>0.9806</td>
</tr>
<tr>
<td>TSLA</td>
<td>2.356797</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table 2 indicates that there is no stationary data. Therefore, differencing is carried out before estimating the model parameters to get the stationary one.

First, the model parameters can be estimated after obtaining stationary data through the differencing process. For the best ARIMA-GARCH model based on AIC, the significant parameter model of each stock are stated in Table 3. According to formula (5) and (8). The full model can be written as

- AAPL

\[
w_t = 0.050904 - 0.602387w_{t-1} + 0.152217w_{t-2} + \alpha_t - 0.728637\alpha_{t-1}
\]

\[
\sigma_t^2 = 1.039583 + 0.289830\alpha_{t-1}^2
\]

- MSFT

\[
w_t = 0.103251 - 0.956880w_{t-1} - 0.774892w_{t-2} - 0.092009w_{t-3} + \alpha_t - 0.886725\alpha_{t-1} - 0.680193\alpha_{t-2}
\]

\[
\sigma_t^2 = 3.099956 + 0.15\alpha_{t-1}^2 + 0.05\alpha_{t-2}^2
\]

- GOOG

\[
w_t = 0.507973 - 0.670364w_{t-1} + 0.064923 + \alpha_t - 0.704250\alpha_{t-1}
\]

\[
\sigma_t^2 = 2.265719 + 0.739152\alpha_{t-1}^2
\]

- AMZN

\[
w_t = 1.085834 - 1.254674w_{t-1} - 0.853132w_{t-2} + \alpha_t - 1.320794\alpha_{t-1} - 0.902022\alpha_{t-2}
\]

\[
\sigma_t^2 = 5.989786 + 0.812942\alpha_{t-1}^2
\]

- TSLA

\[
w_t = 0.062952 - 0.244092w_{t-1} + 0.102440w_{t-2} + \alpha_t - 0.162214\alpha_{t-1}
\]

\[
\sigma_t^2 = 7.151434 + 1.5004\alpha_{t-1}^2
\]
Table 3. Estimated Parameter of The Best ARIMA-GARCH Model for Each Stock

<table>
<thead>
<tr>
<th>Stock</th>
<th>ARIMA-GARCH model</th>
<th>Parameter</th>
<th>Parameter Coefficient</th>
<th>P-Value</th>
<th>Parameter</th>
<th>Parameter Coefficient</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>ARIMA (2,1,1)</td>
<td>Constant</td>
<td>0.050904</td>
<td>0.0493</td>
<td>Constant</td>
<td>1.039583</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>-GARCH (0,1)</td>
<td>AR(1)</td>
<td>-0.602387</td>
<td>0.0000</td>
<td>RESID2(1)</td>
<td>0.289830</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AR(2)</td>
<td>0.152217</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA(1)</td>
<td>0.728637</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSFT</td>
<td>ARIMA (3,1,2)</td>
<td>Constant</td>
<td>0.103251</td>
<td>0.0082</td>
<td>Constant</td>
<td>3.099956</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>-GARCH (0,2)</td>
<td>AR(1)</td>
<td>-0.956880</td>
<td>0.0000</td>
<td>RESID2(1)</td>
<td>0.150000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AR(2)</td>
<td>-0.774892</td>
<td>0.0000</td>
<td>RESID2(2)</td>
<td>0.050000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AR(3)</td>
<td>-0.092009</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA(1)</td>
<td>0.886725</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA(2)</td>
<td>0.680193</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOOG</td>
<td>ARIMA (2,1,1)</td>
<td>Constant</td>
<td>0.507973</td>
<td>0.0774</td>
<td>Constant</td>
<td>2.265719</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>-GARCH (0,1)</td>
<td>AR(1)</td>
<td>-0.670364</td>
<td>0.0000</td>
<td>RESID2(1)</td>
<td>0.739152</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AR(2)</td>
<td>0.064923</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA(1)</td>
<td>0.704250</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AMZN</td>
<td>ARIMA (2,1,2)</td>
<td>Constant</td>
<td>1.085834</td>
<td>0.0430</td>
<td>Constant</td>
<td>5.989786</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>-GARCH (0,1)</td>
<td>AR(1)</td>
<td>-1.254674</td>
<td>0.0000</td>
<td>RESID2(1)</td>
<td>0.812942</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AR(2)</td>
<td>-0.853132</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA(1)</td>
<td>1.320794</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA(2)</td>
<td>0.902022</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TSLA</td>
<td>ARIMA (2,1,1)</td>
<td>Constant</td>
<td>0.062952</td>
<td>0.7460</td>
<td>Constant</td>
<td>7.151434</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>-GARCH (0,1)</td>
<td>AR(1)</td>
<td>-0.244902</td>
<td>0.0000</td>
<td>RESID2(1)</td>
<td>1.500400</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AR(2)</td>
<td>0.102440</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MA(1)</td>
<td>0.162214</td>
<td>0.0028</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Optimized Portfolio

Portfolio optimization is obtained after the information on the mean return of each stock is known. The estimate of the mean stock return is obtained based on the ARIMA-GARCH model that has been formed, which is listed in Table 4.

Table 4. Estimated Mean Return

<table>
<thead>
<tr>
<th>Stock</th>
<th>Mean Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>0.000967466</td>
</tr>
<tr>
<td>MSFT</td>
<td>0.000936567</td>
</tr>
<tr>
<td>GOOG</td>
<td>0.000171679</td>
</tr>
<tr>
<td>AMZN</td>
<td>0.001068245</td>
</tr>
<tr>
<td>TSLA</td>
<td>0.001403081</td>
</tr>
</tbody>
</table>

Based on Table 4, information is obtained that TSLA has the greatest mean return with a 0.14% value. In the Multi-Objective method, the formation of each asset’s weight is determined through the mean return and by considering risk or volatility. In addition, this method also considers the risk aversion value of investors. The greater investors’ risk-aversion indicates that investors tend to maintain safe conditions to avoid investment risks that cause high losses. The movement of the results of calculating the weight of each stock in the optimized portfolio based on the risk aversion value is stated in Figure 2.
Figure 2. Portfolio Weight Based on Risk Aversion Value

We assume that an investor does not want to incur losses of more than USD 100 (USD 100 risk aversion) on this portfolio investment. Then the weight of each stock is obtained for the optimized portfolio, which is stated in Table 5.

Table 5. Weight of Each Stock in Portfolio

<table>
<thead>
<tr>
<th>Stock</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>0.231769937</td>
</tr>
<tr>
<td>MSFT</td>
<td>0.311375499</td>
</tr>
<tr>
<td>GOOG</td>
<td>0.311218321</td>
</tr>
<tr>
<td>AMZN</td>
<td>0.141717391</td>
</tr>
<tr>
<td>TSLA</td>
<td>0.003918852</td>
</tr>
</tbody>
</table>

Based on the table above, MSFT shares have the most considerable weight in the portfolio, whereas TSLA shares have negligible weight. The movement of the optimized portfolio value according to the weight is illustrated according to Figure 3.

Figure 3. Portfolio Price Movement

The process of forming the optimized portfolio weight based on the Multi-Objective method shows that the weight of each
stock is not much different; it is just that TSLA has a relatively minimal weight compared to other stocks. This occurrence could be due to the high volatility in TSLA shares. To clarify the allegations, obtained through the following analysis, measurement of investment risk using Expected Tail Loss (ETL).

4. Hybrid Monte Carlo-Expected Tail Loss

At this stage, the first step is to get a return from the optimized portfolio, not forgetting to calculate the mean and standard deviation. A Monte Carlo simulation was carried out based on the mean and standard deviation of optimized portfolio returns to obtain statistical stability. After just calculating Expected Tail Loss (ETL) using the historical method, namely the movement of past returns. This study uses 1000 iterations to calculate the mean; the ETL value is considered stable. The simulation results for calculating ETL are illustrated according to Figure 4.

![Figure 4. ETL Calculation Simulation Results](image)

According to the picture above, it can be seen that each iteration will produce a different calculation value. Therefore, the average value of these results is considered a reasonable and stable value to measure the investment risk of the optimized portfolio.

After the 1000th iteration, ETL's mean or average value is in the range of 0.029. From these results, it can be concluded that the ETL value of the optimized portfolio is 2.9%. This value shows the maximum expected loss experienced by investors the next day after the research period. In other words, suppose an investor invests US$ 1000 of capital, then with a five percent probability, the total expected daily loss in the optimized portfolio will equal or exceed US$ 29 (2.9% of US$ 1000).

After getting the ETL value from the optimized portfolio, we will then compare it with the ETL value of each stock. The comparison results are displayed as shown in Figure 5. Based on the illustration in the figure, it can be seen that the optimized portfolio has the lowest ETL value. These results conclude that investing in an optimized portfolio provides more security for losses arising from investment risk. Thus, this study proves that investment risk modeling and analysis can obtain an optimal portfolio that provides the lowest level of risk so that the expected losses that investors can experience can be minimized.
E. CONCLUSION AND SUGGESTION

Based on the results of modeling and analysis of the optimized portfolio of US big-cap Stocks based on Multi-Objective Optimization (MOO) using Autoregressive Integrated Moving Average-Generalized Autoregressive Conditional Heteroscedasticity (ARIMA-GARCH) and Monte Carlo-Expected Tail Loss (MC-ETL), several conclusions can be drawn. There are 3 variations of the ARIMA-GARCH model, including ARIMA(2,1,1)-GARCH(0,1) for AAPL, GOOG, and TSLA, ARIMA(3,1,2)-GARCH(0,2) for MSFT, ARIMA(2,1,2)-GARCH(0,1) for AMZN. Based on this model, TSLA has the highest mean return, while GOOG has the lowest mean return. Calculation of optimized portfolio weight based on MOO has been obtained, and the assumption of risk aversion value is taken to obtain a static optimized portfolio weight. Then the optimized portfolio return volatility is analyzed using ETL to obtain a measure of investment risk. The Monte Carlo simulation on the ETL calculation yields a value of 2.9% for the optimized portfolio. This is the smallest value compared to all the constituent stocks’ ETL values. Thus, the optimized portfolio is concluded as an investment choice for investors with a low level of risk.

For the next research, we suggest using other models to model stock prices. Portfolio formation methods and other risk measurements can also be used to obtain comparisons. Finally, use the calculation of the level of accuracy to find out how much the model can estimate the actual data.

REFERENCES


