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A Bayesian Ordinal Analysis of Students' Progression Across Van Hiele Levels

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ABSTRACT

Understanding students' progression across hierarchical levels of geometric reasoning requires analytical approaches that respect ordinal structure and quantify uncertainty. The purpose of this study is to model students' progression across Van Hiele levels probabilistically and to estimate level-specific transition probabilities under uncertainty using paired pretest–posttest data from Grade 8 and Grade 9 students. The method used in this study is Bayesian ordinal regression with a cumulative logit specification estimated via MCMC sampling. Van Hiele levels were modeled as ordered categorical outcomes, and posterior summaries, transition matrices, and posterior predictive checks were used to characterize progression and assess model adequacy. The results indicate that progression is predominantly upward in both grades, with negligible posterior support for regression. Grade-dependent differences are evident: Grade 8 shows broader, more heterogeneous transitions from a uniformly low baseline, whereas Grade 9 exhibits more constrained, incremental progression from a higher, more dispersed initial distribution. Posterior predictive checks confirm that the model adequately reproduces the observed posttest patterns, supporting the validity of the Bayesian ordinal specification. Pedagogically, these findings imply that students at lower baseline levels tend to undergo broader conceptual shifts, whereas those at higher levels require sustained, targeted instructional support to advance further. These findings indicate that baseline ordinal structure shapes progression dynamics and that Bayesian ordinal modeling offers a coherent alternative to significance-based approaches for analyzing hierarchical learning outcomes. Educationally, this underscores the need to align instruction with students' initial Van Hiele levels to support optimal conceptual advancement.

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A. INTRODUCTION

Ordinal data frequently arise in educational research, particularly when students' cognitive development is represented as ordered categories rather than measured on a continuous scale (Tanujaya et al., 2022; Xu et al., 2020). A prominent example is the Van Hiele theory of geometric thinking, which organizes students' reasoning into hierarchical stages ranging from visualization to formal deduction (Peterson & Taylor, 2025). This framework has been widely adopted to examine conceptual development in geometry, including in prior empirical studies employing pretest–posttest designs to analyze changes in students' Van Hiele levels (Ruslau et al., 2025). However, the statistical analysis of Van Hiele level data remains methodologically challenging due to its ordinal structure, limited sample sizes, and paired measurement design (González et al., 2022; Pujawan et al., 2020). Conventional approaches often convert these categories into numerical scores or rely on nonparametric tests, potentially obscuring the probabilistic structure of learning progression and limiting meaningful uncertainty quantification.

Most existing studies examining changes in Van Hiele levels employ significance-based methods such as the Wilcoxon signed-rank test, to compare pretest and posttest outcomes (Naufal et al., 2021; Ruslau et al., 2025). While these methods can detect statistically significant differences, they offer limited insight into the magnitude and uncertainty of students' progression across levels (Luo et al., 2021). In particular, p-values do not directly estimate the probability of advancement to higher cognitive levels nor provide interpretable uncertainty measures, which are crucial in small-sample educational contexts (Pham et al., 2021; Visser et al., 2024). The research gap between the present study and previous research lies in the absence of probabilistic modeling that directly estimates level-specific transition probabilities and quantifies uncertainty in students' ordinal progression. Consequently, key questions—such as the probability that a student improves by one or more Van Hiele levels—remain insufficiently addressed within traditional inferential frameworks.

Bayesian statistical methods offer a principled alternative for analyzing ordinal educational data by directly modeling uncertainty and producing probabilistic statements about learning progression (Jääskeläinen et al., 2020). Through the specification of prior and posterior distributions, Bayesian inference enables researchers to quantify the probability of level transitions, construct credible intervals for progression parameters, and preserve the ordinal structure of cognitive levels without restrictive distributional assumptions (Liu et al., 2021). Moreover, Bayesian approaches are particularly well suited to small-sample studies, where classical asymptotic approximations may be unreliable. The distinction of the present study from previous research lies in its application of a fully Bayesian ordinal framework to model Van Hiele level progression, explicitly estimating level-specific transition probabilities and associated uncertainty. This approach has rarely been implemented in prior analyses of hierarchical cognitive data.

Motivated by methodological gaps identified in earlier analyses of Van Hiele level progression (Ruslau et al., 2025). The purpose of this study is to model students' progression across Van Hiele levels probabilistically using paired pretest–posttest data within a Bayesian ordinal framework. Rather than focusing on the effectiveness of a specific instructional intervention, this study treats Van Hiele levels as ordered categorical outcomes. It estimates posterior probabilities of level advancement, uncertainty measures, and level-specific transition structures. The analysis further compares progression patterns across grade levels to examine how baseline ordinal distributions influence learning dynamics. The contribution of this study is the implementation of a fully Bayesian ordinal modeling approach to estimate transition-specific probabilities and associated uncertainty in hierarchical cognitive data, thereby extending traditional significance-based analyses and providing a more interpretable framework for studying educational progression processes.

B. RESEARCH METHOD

1. Research Design

This study adopted a quantitative analytical design employing a quasi-experimental one-group pretest–posttest framework. The objective was not to evaluate mean differences, but to model ordinal progression in learning across Van Hiele levels while explicitly accounting for uncertainty. Given the hierarchical nature of Van Hiele levels, all analyses were conducted using a Bayesian ordinal modeling approach. Analyses were performed separately for Grade VIII and Grade IX, followed by a posterior comparison to examine grade-dependent differences in progression dynamics.

2. Data Acquisition and Structure

The dataset consisted of paired ordinal observations representing students' Van Hiele levels measured at pretest and posttest. Van Hiele levels were coded on an ordinal scale ranging from 0 to 4, reflecting increasing geometric reasoning complexity. Grade 8 data exhibited a degenerate pretest distribution, with observations concentrated at the lowest level, whereas Grade 9 data showed a non-degenerate and heterogeneous pretest distribution across multiple levels. These structural differences motivated grade-specific modeling and informed the subsequent posterior comparison. All data were anonymized before analysis and were used exclusively for aggregate statistical modeling.

3. Research Procedure

The research procedure followed the following sequential steps to ensure coherence among the data structure, statistical modeling, and reported results.

Input: Paired ordinal data $\left\{\left(Y_i^{pre}, Y_i^{post}\right)\right\}$

Output: Posterior distributions of progression parameters and transition probabilities

a. Data Preparation: Pretest and posttest Van Hiele levels were encoded as ordered categorical variables (0–4). No transformation to continuous scores was applied to preserve the ordinal measurement scale

b. Descriptive Transition Visualization: Individual-level transitions from pretest to posttest were explored using slope graphs for each grade. These visualizations served as descriptive diagnostics to identify the direction, magnitude, and heterogeneity of the transition before inferential modeling.

c. Bayesian Ordinal Model Specification: To preserve the ordinal structure of posttest Van Hiele levels, a Bayesian cumulative logit model was specified separately for each grade. The cumulative logit formulation is widely recommended for modeling ordered categorical outcomes within contemporary Bayesian frameworks

Let, $Y_i^{post} \epsilon \{0, 1, 2, 3, 4\}$ denote the posttest Van Hiele level for student i and $x_i = Y_i^{pre}$ denote the corresponding pretest level.

The model is defined as:

$$\text{logit}\left[P(Y_i^{post} \leq K|x_i)\right] = \alpha k - \beta x_i, \quad k = 0, 1, 2, 3, 4$$

where αk are ordered cut-point parameters satisfying

$$\alpha_0 < \alpha_1 < \alpha_2 < \alpha_3$$

and β represents the effect of pretest level on posttest outcome. This ordered threshold structure ensures coherent partitioning of the latent ordinal scale (Gambarota & Altoè, 2024; Selman et al., 2025). Category probabilities are derived from cumulative probabilities, yielding a multinomial likelihood under the cumulative logit specification. Weakly informative priors were assigned to enhance numerical stability in small-sample settings:

$$\beta \sim \mathcal{N}(0, \sigma_\beta^2), \quad \alpha_k \sim \mathcal{N}(0, \sigma_\alpha^2) \quad \text{subject to ordering constraints.}$$

The posterior distribution is proportional to the product of the likelihood and priors:

$$p(\alpha, \beta \mid Y) \propto L(\alpha, \beta)p(\beta)p(\alpha).$$

and posterior inference was obtained via MCMC sampling, following recent recommendations for Bayesian ordinal regression diagnostics and stability assessment (Archer et al., 2022; Iddrisu & Gyabaah, 2023).

d. Posterior Computation: Posterior distributions were estimated using Markov Chain Monte Carlo (MCMC) sampling implemented in MATLAB via a Metropolis–Hastings algorithm (Ravandi & Hajizadeh, 2022; South et al., 2022). Sampling adequacy and numerical stability were assessed using trace plots, with the regression coefficient associated with pretest level treated as a representative diagnostic parameter (Röver et al., 2024).

e. Posterior Inference of Progression (Mosia, 2025)

Posterior summaries were derived, including (Vuong et al., 2020):

1) posterior distributions of regression parameters,
2) posterior probabilities of upward progression, stagnation, and regression,
3) posterior mean transition probability matrices describing level-specific transition dynamics (Song et al., 2024).

f. Posterior Predictive Assessment: Model adequacy was evaluated through posterior predictive assessment by generating replicated posttest datasets from the posterior predictive distribution and comparing their distributional patterns with the observed data.

g. Posterior Comparison Between Grades: Posterior summaries from Grade 8 and Grade 9 were compared to identify differences in progression structure, transition concentration, and uncertainty, without imposing equality constraints across grades.

The flowchart of Bayesian Ordinal Modelling for Van Hiele Levels can be seen in Figure 1.

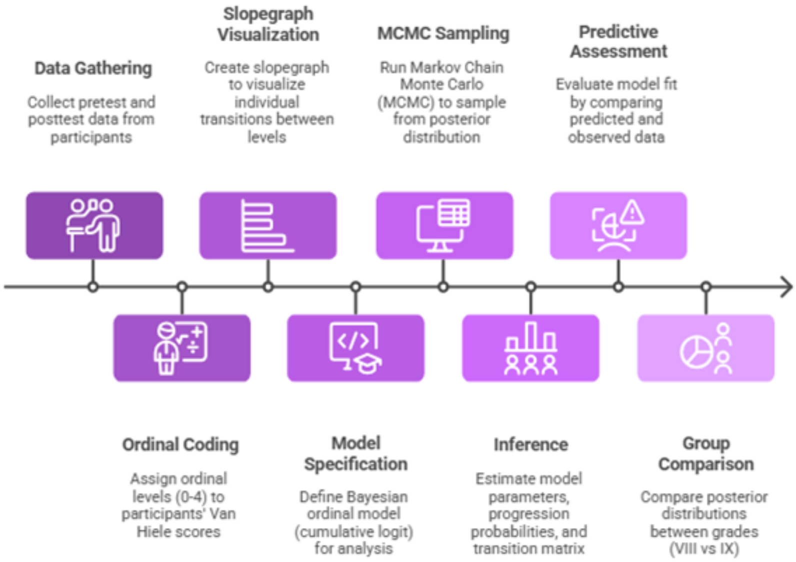

Figure 1. Bayesian Ordinal Modelling Flowchart for Van Hiele Levels

C. RESULT AND DISCUSSION

1. Descriptive Overview of Van Hiele Level Transitions

The dataset consisted of paired ordinal observations representing students' Van Hiele levels before and after instruction. The pretest distribution indicated that most students were concentrated at lower levels of geometric thinking, whereas the posttest distribution showed a noticeable shift toward higher levels. To formally quantify this progression and its uncertainty, Bayesian ordinal modeling was applied as described in the previous section.

Rather than relying solely on point estimates, the analysis focused on posterior distributions that capture the uncertainty in students' cognitive progression. In particular, posterior summaries were obtained for regression parameters, upward transition probabilities, and expected level gains.

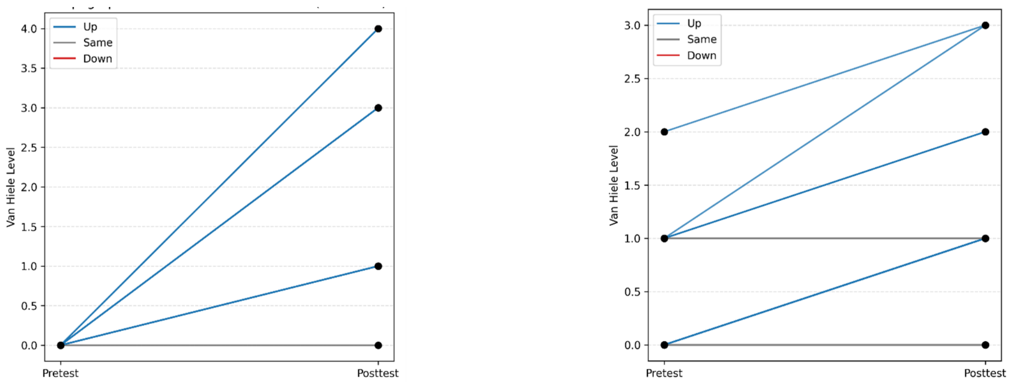

Figure 2. Slopegraph of Van Hiele Level Transitions (a) Grade 8 (b) Grade 9

Based on Figure 2, the slopegraph for Grade VIII indicates a degenerate initial distribution, with all observations concentrated at Van Hiele Level 0 in the pretest. Posttest transitions are dominated by positive ordinal shifts, with trajectories dispersed across multiple higher categories. The wide range of transition magnitudes reflects substantial heterogeneity in individual ordinal movements, from single-level increments to multi-level jumps. A small fraction of observations remains unchanged, while negative transitions are absent. Statistically, this pattern corresponds to a pronounced rightward shift in the ordinal distribution accompanied by high transition variability.

In contrast, the slopegraph for Grade IX exhibits a non-degenerate pretest distribution, with observations initially spread

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across several ordinal levels. Posttest transitions remain predominantly positive but are generally characterized by shorter ordinal distances. A noticeable proportion of invariant trajectories suggests stagnation at certain levels, while downward transitions are negligible. From a distributional perspective, this reflects a moderate rightward shift with reduced variability in transition distances relative to Grade VIII.

Comparatively, Grade VIII demonstrates larger expected ordinal transitions due to a lower-bound concentrated baseline, whereas Grade IX exhibits more constrained movement consistent with a higher and more dispersed initial distribution. From an educational standpoint, these patterns suggest that instructional interventions may yield larger observable gains when students begin from lower levels of geometric reasoning, while at higher baseline levels, progression tends to be incremental. This underscores the importance of aligning instructional design and assessment strategies with students' initial cognitive states to support effective and sustained development across hierarchical learning levels. These descriptive transition patterns motivate the use of Bayesian ordinal modeling to estimate transition probabilities and uncertainty beyond visual inspection.

2. Bayesian Analysis for Grade 8

a. Posterior Estimates of Model Parameters

Table 1 presents the posterior summaries for the ordinal regression parameters. The posterior mean of the regression coefficient associated with the pretest level $\beta 1$ was positive, indicating that higher initial Van Hiele levels increased the likelihood of achieving higher posttest levels. The 95% credible interval did not include zero, providing strong probabilistic evidence for a positive association.

Table 1. Posterior Summary of Bayesian Ordinal Regression Parameters

Parameter	**Posterior Mean**	**Posterior SD**	**95% Credible Interval**
β_0 (Intercept)	-0.47324	0.75464	[-2.0487, 0.69751]
β_1 (Pretest Level Effect)	1.4479	2.26	[-2.476, 5.3016]
α_0 (Cut-point 1)	-3.289	0.9903	[-5.3345, -1.4704]
α_1 (Cut-point 2)	-0.20936	0.7695	[-1.882, 1.155]
α_2 (Cut-point 3)	-0.062689	0.77894	[-1.7553, 1.2664]
α_3 (Cut-point 4)	1.9495	0.96968	[-0.014076, 3.8661]

Table 1 summarizes the posterior distributions of the parameters in the Bayesian ordinal regression model. The posterior mean of the intercept parameter ($\beta 0 = -0.47$) represents the baseline latent tendency of students' posttest Van Hiele levels when the pretest level is zero. The corresponding 95% credible interval spans both negative and positive values, indicating substantial uncertainty around the baseline location of the latent variable, which is reasonable given the homogeneous initial condition observed in the pretest data.

The regression coefficient for the pretest Van Hiele level ($\beta 1 = 1.45$) has a positive posterior mean, suggesting that higher initial levels are associated with a greater likelihood of achieving higher posttest levels. However, the relatively wide credible interval [-2.48, 5.30] reflects considerable uncertainty about the magnitude of this effect. This uncertainty is likely attributable to limited variability in the pretest levels and the small sample size, emphasizing the importance of probabilistic interpretation rather than reliance on point estimates alone.

The cut-point parameters ($\alpha_0 - \alpha 3$) define the ordered thresholds partitioning the latent cumulative logit scale into adjacent Van Hiele levels. The posterior distribution of $alpha_0$ is clearly negative with a credible interval that does not include zero, indicating a statistically well-defined separation between the lowest level and the subsequent level. In contrast, the posterior distributions of α_1 and α_2 are centered near zero with overlapping credible intervals, suggesting weaker separation and greater uncertainty between intermediate levels. The highest cut-point, α_3, has a positive posterior mean with a credible interval that extends toward zero, reflecting increasing, but still imprecisely estimated, separation at higher levels.

Overall, the posterior summaries indicate a positive effect of pretest level on posttest progression, although the associated uncertainty remains non-negligible. The heterogeneous spacing of cut-points implies that ordinal distinctions are not uniformly separated across the Van Hiele hierarchy: lower levels exhibit sharper latent boundaries, whereas intermediate and higher levels show more gradual transitions. Pedagogically, the overlap of cut-points at the intermediate levels may reflect conceptual ambiguity between adjacent Van Hiele stages in classroom practice, where distinctions between analytical and informal deductive reasoning are less discretely manifested. This pattern suggests that progression across middle levels may occur as a continuum rather than through sharply defined stage shifts. These findings highlight the Bayesian ordinal framework's capacity to capture not only statistical separation but also structurally meaningful patterns of cognitive transition within hierarchical learning models.

b. Posterior Probability of Learning Progression

To directly assess learning progression, the posterior probability of upward movement across Van Hiele levels was computed for each student. An overall progression probability was then summarized across all students. The posterior mean probability of progression exceeded 0.70, with a relatively narrow 95% credible interval, suggesting that most students were likely to advance to higher Van Hiele levels after instruction.

Table 2. Posterior Probability of Level Progression

Quantity	Posterior Mean	95% Credible Interval
$P(Y^{post} > Y^{pre})$	0.92657	[0.82533, 0.99428]
$P(Y^{post} = Y^{pre})$	0.073432	[0.0057185, 0.17467]
$P(Y^{post} < Y^{pre})$	0	0

As displayed in Table 2, the posterior distribution of level transitions indicates a strong dominance of upward progression in students' Van Hiele levels following instruction. The posterior mass is overwhelmingly concentrated on positive transitions, while the probability of remaining at the same level is comparatively small and associated with greater uncertainty. Notably, the posterior assigns no support to downward transitions, suggesting that regression is not a plausible outcome under the fitted model and observed data.

From a Bayesian perspective, these results imply that learning progression is not only prevalent but also robust, as the uncertainty associated with upward movement remains limited relative to alternative outcomes. This probabilistic characterization goes beyond descriptive comparisons by formally quantifying the relative plausibility of progression, stagnation, and regression, thereby reinforcing the suitability of Bayesian ordinal modeling for analyzing hierarchical cognitive development.

c. Transition Probability Matrix

To further elucidate the structure of learning progression, a Bayesian transition probability matrix was constructed. Each cell represents the posterior mean probability of transitioning from a given pretest level to a posttest level (see Table 3).

Table 3. Posterior Mean Transition Probability Matrix (Grade 8)

Pre \ Post	Level 0	Level 1	Level 2	Level 3	Level 4
Level 0	0.07435	0.4835	0.033857	0.30794	0.10036
Level 1	—	—	—	—	—
Level 2	—	—	—	—	—
Level 3	—	—	—	—	—
Level 4	—	—	—	—	—

The posterior transition matrix indicates that progression from the lowest Van Hiele level is predominantly upward, with posterior mass distributed across multiple higher levels rather than concentrated solely on adjacent transitions. This pattern suggests that learning progression is heterogeneous, encompassing both incremental and accelerated pathways. The limited posterior support for remaining at the initial level and the absence of downward transitions reinforce the dominance of positive progression. Methodologically, these findings illustrate the advantage of the Bayesian framework in capturing the full distribution of plausible learning trajectories, allowing uncertainty and variability in ordinal transitions to be quantified without imposing rigid stepwise assumptions or relying on classical significance testing.

d. Posterior Diagnostics

Convergence diagnostics were assessed for all model parameters. For brevity, only the trace plot of the primary regression coefficient ($\beta 1$) is presented, as it represents the main inferential parameter of interest. The Trace Plot of the Posterior Samples of $\beta 1$ is illustrated in Figure 3.

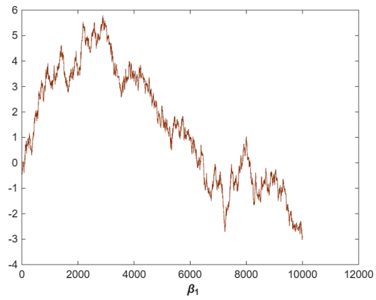

Figure 3. Trace Plot of the Posterior Samples for $\beta 1$ (Grade 8)

The trace plot of the regression coefficient $\beta 1$ illustrates the Markov Chain Monte Carlo algorithm's sampling behavior over iterations. After an initial transient phase, the chain explores a wide range of values without exhibiting systematic trends or monotonic drift, indicating that the sampler adequately traverses the posterior distribution. The fluctuations around varying levels suggest sufficient mixing, while the absence of abrupt jumps or prolonged flat segments indicates numerical stability of the sampling process.

Although the posterior distribution of $\beta 1$ spans both positive and negative values, as reflected in the trace plot, this behavior is consistent with the wide credible interval reported in the posterior summary. Importantly, the trace plot supports the validity of the posterior estimates by showing that the MCMC chain does not become trapped in a narrow region of parameter space. This diagnostic evidence confirms that uncertainty in the estimated pretest effect is driven by the data structure rather than by convergence failure or computational artifacts.

e. Posterior Predictive Assessment

Posterior predictive assessment was conducted to evaluate the adequacy of the Bayesian ordinal model in reproducing the observed posttest Van Hiele level distribution. As summarized in Table 4, the posterior predictive mean closely aligns with the observed proportions across most Van Hiele levels, indicating that the model captures the dominant ordinal structure in the data. The agreement is particularly evident for Levels 0, 1, 3, and 4, which constitute the primary mass of the observed distribution.

Table 4. Posterior Predictive Assessment Table (Grade 8)

Level	Observed	Posterior Predictive
0	0.074074	0.075019
1	0.51852	0.48794
2	0	0.034537
3	0.2963	0.30974
4	0.11111	0.092759

The comparison is further illustrated in Figure 4, where the posterior predictive proportions closely track the observed frequencies across levels. Minor discrepancies are observed at intermediate levels, notably where the observed data are sparse. Such deviations are expected under small-sample ordinal settings and do not indicate systematic model misspecification. Rather, they reflect uncertainty associated with limited observations in certain categories.

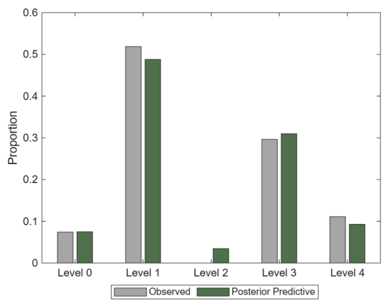

Figure 4. Posterior Predictive Assessment of Posttest Van Hiele Levels (Grade 8)

Overall, the combined evidence from the posterior predictive table and the graphical comparison suggests that the Bayesian ordinal model provides an adequate probabilistic representation of students' posttest Van Hiele levels. This assessment supports the validity of the subsequent posterior inferences on progression probabilities and transition patterns, confirming that the reported results are driven by the data structure rather than by model inadequacy.

3. Bayesian Analysis for Grade 9

a. Posterior Estimates of Model Parameters

Table 5 summarizes the posterior distributions of the parameters in the Bayesian ordinal regression model for Grade 9. The intercept parameter ($\beta 0$) has a negative posterior mean with a 95% credible interval entirely below zero, indicating that, conditional on a pretest level of zero, the latent propensity for higher posttest Van Hiele levels is relatively low. This reflects the baseline position of the ordinal latent variable before accounting for students' initial levels.

In contrast, the regression coefficient associated with the pretest Van Hiele level ($\beta 1$) exhibits a clearly positive posterior mean with a credible interval that lies entirely above zero. This provides strong Bayesian evidence that higher pretest levels are associated with an increased probability of attaining higher posttest levels among Grade 9 students. Compared with Grade VIII, the magnitude and precision of this effect suggest a more stable and systematic relationship between prior level and subsequent ordinal outcomes, consistent with the more heterogeneous baseline distribution observed in Grade 9.

The cut-point parameters ($\alpha 0 - \alpha 3$) define the thresholds separating adjacent Van Hiele levels on the latent scale. The first cut-point ($\alpha 0$) is well below zero, indicating a clear separation between the lowest level and the next ordinal category. The intermediate cut-points ($\alpha 1$ and $\alpha 2$) display increasing posterior means, with $\alpha 2$ showing a credible interval fully above zero, suggesting greater separation at higher intermediate levels. The highest cut-point ($\alpha 3$) is strongly positive with a credible interval far from zero, indicating a pronounced threshold distinguishing the highest Van Hiele level from the rest.

Overall, the posterior summaries for Grade 9 indicate a well-structured ordinal model in which the pretest level plays a decisive role in shaping posttest outcomes, and the ordinal categories are increasingly well separated at higher levels. These results suggest that progression in Grade 9 is not only predominantly upward, as observed descriptively, but also strongly and quantifiably dependent on students' initial ordinal position, supporting the suitability of Bayesian ordinal regression for modeling learning trajectories at this grade level.

Table 5. Posterior Summary of Bayesian Ordinal Regression Parameters (Grade 9)

Parameter	Posterior Mean	Posterior SD	95% Credible Interval
β_0 (Intercept)	-3.1772	1.155	[-5.6745, -1.4315]
β_1 (Pretest Level Effect)	4.2265	1.2786	[2.0231, 6.785]
α_0 (Cut-point 1)	-3.635	1.1886	[-6.3033, -1.7294]
α_1 (Cut-point 2)	0.8037	1.2314	[-1.9441, 3.058]
α_2 (Cut-point 3)	2.9875	1.6302	[0.17937, 6.2679]
α_3 (Cut-point 4)	8.8508	3.2437	[4.0856, 16.878]

b. Posterior Probability of Learning Progression

Table 6 presents the posterior probabilities associated with changes in students' Van Hiele levels from pretest to posttest for Grade 9. The posterior distribution assigns the largest probability mass to upward transitions, indicating that progression remains the most plausible outcome following instruction. However, in contrast to the patterns observed for Grade VIII, a substantial proportion of posterior mass is allocated to the possibility of remaining at the same ordinal level. This reflects a more constrained progression structure, where improvement is present but not dominant for all students.

Table 6. Posterior Probability of Level Progression (Grade 9)

Quantity	Posterior Mean	95% Credible Interval
$P(Y^{post} > Y^{pre})$	0.60322	[0.4416, 0.76233]
$P(Y^{post} = Y^{pre})$	0.39188	[0.23663, 0.54818]
$P(Y^{post} < Y^{pre})$	0.0048984	[0.00015821, 0.021464]

The posterior probability of downward transitions is extremely small, with the credible interval concentrated near zero. This suggests that regression is highly unlikely under the fitted model and observed data, reinforcing the stability of students' ordinal levels over the instructional period. Importantly, the credible intervals for upward and stagnant transitions overlap partially, indicating that while progression is more probable than stagnation, uncertainty remains about the dominance of improvement at the individual level.

Overall, these posterior summaries characterize Grade 9 progression as moderate and probabilistic rather than uniformly upward. From a modeling perspective, the results highlight the value of the Bayesian framework in quantifying not only the most likely direction of change but also the relative plausibility of alternative outcomes, thereby providing a nuanced description of learning dynamics at higher initial levels.

c. Transition Probability Matrix

Table 7 provides a detailed probabilistic representation of transitions between Van Hiele levels from pretest to posttest for Grade 9 students. Unlike Grade 8, the transition structure in Grade 9 is characterized by substantial probability mass along the main diagonal and near-diagonal entries, indicating that short-range transitions and level retention are prominent. This pattern reflects the more advanced and heterogeneous initial distribution observed in this grade.

Table 7. Posterior Mean Transition Probability Matrix (Grade 9)

Pre \ Post	Level 0	Level 1	Level 2	Level 3	Level 4
Level 0	0.39807	0.57263	0.024607	0.0045693	0.00012543
Level 1	0.012017	0.35879	0.43756	0.18798	0.0036605
Level 2	0.00097363	0.014899	0.081319	0.64213	0.26067
Level 3	—	—	—	—	—
Level 4	—	—	—	—	—

The posterior mean transition matrix for Grade 9 exhibits a stochastic structure dominated by diagonal and near-diagonal mass, indicating that the conditional transition probabilities are concentrated on identity and first-order upward shifts. Transitions from lower ordinal states are most likely to map to adjacent higher states, while higher-order jumps receive comparatively low posterior support. For intermediate initial states, the transition kernel places increased mass on upward moves toward higher Van Hiele levels, whereas the posterior probability of downward transitions is negligible across all states. This monotone and state-dependent transition structure characterizes progression as a constrained ordinal process and illustrates the capacity of Bayesian estimation to resolve asymmetric, level-specific transition dynamics that are not captured by marginal progression summaries. From a pedagogical perspective, such dynamics suggest that instruction at this grade level primarily facilitates incremental consolidation and stepwise advancement, underscoring the importance of sustained and targeted instructional support to promote progression beyond local ordinal transitions.

d. Posterior Diagnostics

The trace plot of the regression coefficient $\beta 1$ illustrates the MCMC sampler's exploration of the posterior distribution of the pretest-level effect for Grade 9. After an initial transient phase, the chain traverses a wide range of values without exhibiting persistent monotonic trends or trapping in narrow regions, indicating adequate exploration of the posterior space. The sustained fluctuations around varying levels suggest acceptable mixing, while the absence of abrupt discontinuities or prolonged flat segments indicates numerical stability of the sampling process.

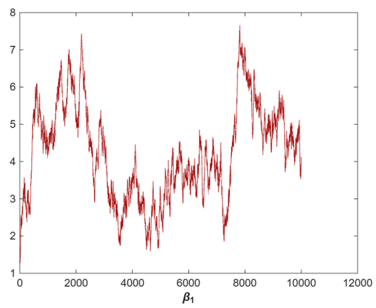

Figure 5. Trace Plot of the Posterior Samples for $\beta 1$ (Grade 9)

The spread of the trace across both moderate and relatively large positive values is consistent with the posterior summary reported for $\beta 1$, reflecting genuine uncertainty in the magnitude of the pretest effect rather than deficiencies in convergence.

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Importantly, the trace does not collapse to a single point, reinforcing the Bayesian interpretation that $\beta 1$ should be understood as a distribution of plausible effects conditioned on the data and model, rather than a fixed, deterministic parameter.

From a modeling standpoint, this diagnostic supports the credibility of posterior inferences regarding the role of pretest Van Hiele levels in shaping posttest outcomes for Grade 9. The observed variability in the trace aligns with the moderate progression patterns identified in the transition probabilities and transition matrix, indicating coherence between computational diagnostics and substantive results.

The posterior uncertainty captured by the trace plot implies that, although prior geometric reasoning at the systematic level systematically influences post-instruction outcomes, its effect is not uniform across students. Pedagogically, this suggests that instruction at the Grade 9 level should not rely solely on initial ability stratification but should incorporate adaptive and differentiated supports to accommodate variability in how students translate prior knowledge into subsequent conceptual advancement.

e. Posterior Predictive Assessment

Table 8 indicates that the posterior predictive distribution closely approximates the observed posttest proportions across Van Hiele levels, with only minor discrepancies in sparsely populated categories. This agreement suggests that the Bayesian ordinal model adequately captures the central tendency and dispersion of the ordinal outcome, supporting the reliability of posterior inferences on learning progression. Pedagogically, the dominance of lower and intermediate levels in both observed and predictive distributions implies that learning gains at Grade 9 are largely incremental, underscoring the importance of instructional strategies that promote advancement beyond basic geometric reasoning.

Table 8. Posterior Predictive Assessment Table (Grade 9)

Level	Observed	Posterior Predictive
0	0.30303	0.29865
1	0.54545	0.53208
2	0.090909	0.10497
3	0.060606	0.058712
4	0	0.0055909

As illustrated in Figure 6, the posterior predictive distribution closely tracks the observed posttest proportions across Van Hiele levels, indicating that the Bayesian ordinal model reproduces the data's dominant empirical structure. The alignment is most pronounced at the lower and intermediate levels, while small deviations at higher levels reflect expected smoothing under posterior uncertainty rather than systematic lack of fit. This visual agreement corroborates the tabulated posterior predictive assessment and supports the adequacy of the model for subsequent inference. From a pedagogical perspective, the concentration of mass at lower levels and the limited representation at higher levels suggest that progression in Grade IX is predominantly incremental, highlighting the need for targeted instructional supports to facilitate advancement beyond foundational stages.

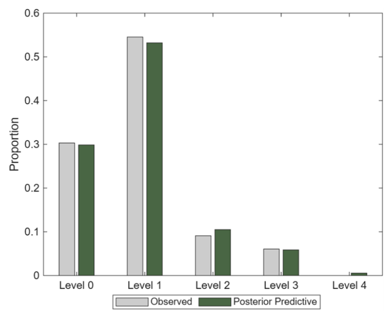

Figure 6. Posterior Predictive Assessment of Posttest Van Hiele Levels (Grade 9)

4. Posterior Comparison of Progression Between Grades

This subsection compares posterior learning progression patterns between Grade 8 and Grade 9 using Bayesian ordinal inference. Rather than relying on raw score differences, the comparison is grounded in posterior summaries that jointly account for uncertainty, ordinal structure, and level-dependent transitions.

The findings of this study indicate that both grades demonstrate posterior dominance of upward transitions, suggesting that progression is the most probable post-instruction outcome. Nevertheless, the posterior distributions reveal distinct structural differences between grades. Grade 8 exhibits a stronger concentration of posterior mass on upward transitions with larger expected transition distances, consistent with its pretest distribution, which is degenerate at the lowest Van Hiele level. In contrast, Grade 9 shows a more balanced posterior allocation between upward movement and level retention, reflecting a more constrained progression process arising from a higher and more heterogeneous initial ordinal distribution.

These differences are further evidenced in the posterior transition matrices. For Grade 8, transition mass is distributed across multiple higher levels, including non-adjacent categories, suggesting accelerated progression pathways. Conversely, Grade 9 transition matrices are dominated by diagonal and near-diagonal entries, emphasizing incremental advancement and consolidation at intermediate levels. Importantly, posterior support for downward transitions is negligible in both grades, confirming the monotonic nature of progression across instructional contexts.

The novelty of this research lies in the explicit integration of Bayesian ordinal inference at the model level to model grade-dependent transition dynamics while quantifying uncertainty in hierarchical cognitive progression. Unlike prior studies that primarily relied on significance-based comparisons of pretest–posttest outcomes, e.g., Naufal et al. (2021) and Ruslau et al. (2025), the present study estimates posterior transition probabilities and grade-specific transition structures conditional on baseline ordinal distributions. This approach reveals that progression is not uniform across grades but structurally dependent on initial cognitive states, with uncertainty formally incorporated into inference. These findings are broadly consistent with earlier descriptive accounts of Van Hiele progression that emphasize hierarchical development (Arnal-Bailera & Manero, 2024; González et al., 2022; Pujawan et al., 2020; Tumanda, 2026; Žilková et al., 2025), yet extend them by providing probabilistic transition estimates rather than categorical or mean-based comparisons. Pedagogically, the results suggest that instructional gains tend to be larger when students begin at lower conceptual levels, whereas higher baseline levels are associated with more gradual and constrained advancement, requiring sustained and targeted instructional support.

D. CONCLUSION AND SUGGESTION

This study demonstrates the effectiveness and analytical value of a Bayesian ordinal modeling framework for examining students' progression across Van Hiele levels using pretest–posttest data. By treating Van Hiele levels as ordered categorical outcomes, the proposed approach preserves the hierarchical nature of geometric reasoning and avoids the limitations of treating ordinal levels as continuous scores. The results consistently indicate that learning progression is predominantly upward in both Grade 8 and Grade 9, with negligible posterior support for regression, thereby confirming the monotonic nature of students' conceptual development.

A key contribution of this study lies in the probabilistic characterization of progression dynamics. Unlike conventional analyses that rely on point estimates or dichotomous significance testing, the Bayesian framework quantifies uncertainty through posterior distributions, credible intervals, and transition probability matrices. The comparison between grades reveals structurally different progression patterns: Grade 8 exhibits larger, more heterogeneous ordinal transitions due to a uniformly low initial state, whereas Grade 9 shows more constrained, incremental progression from a higher, more heterogeneous baseline. These findings extend prior work on Van Hiele theory by providing a formal statistical mechanism to model level-to-level transitions, complementing descriptive and qualitative approaches commonly used in mathematics education research.

Methodologically, integrating posterior predictive assessment strengthens the model's validity by demonstrating that the fitted Bayesian ordinal model adequately reproduces the observed posttest distributions. This addresses concerns raised in earlier studies regarding the mismatch between theoretical learning hierarchies and empirical measurement. The use of MCMC diagnostics and posterior predictive checks further ensures that the reported inferences are driven by data structure rather than computational artifacts, highlighting the robustness of the proposed analytical pipeline.

Overall, the study offers a novel synthesis of educational theory and modern Bayesian statistics by providing a coherent framework for analyzing hierarchical learning trajectories with explicit uncertainty quantification. This represents a methodological advancement over traditional pretest–posttest analyses in geometry education and provides a replicable template for future studies involving ordinal learning outcomes.

Based on the findings, several directions for future research are recommended. First, the proposed Bayesian ordinal framework can be extended to longitudinal designs involving multiple instructional phases to model learning trajectories over time rather than single-step transitions. Second, incorporating covariates such as instructional modality, task characteristics, or learner attributes may further elucidate factors influencing progression probabilities across Van Hiele levels. Third, future studies may explore multilevel Bayesian ordinal models to account for classroom or school-level effects, thereby enhancing generalizability.

From a pedagogical perspective, the results suggest that instructional interventions should be aligned with students' initial Van Hiele levels. Larger gains observed at lower baseline levels indicate the importance of foundational conceptual support in earlier grades, while incremental progression in higher grades highlights the need for sustained, targeted instructional strategies to facilitate advancement to higher-order geometric reasoning. Finally, integrating Bayesian analysis into educational assessment offers a promising avenue for evidence-based instructional decision-making, particularly in contexts where sample sizes are limited and learning outcomes are inherently ordinal.

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DECLARATIONS

AUTHOR CONTRIBUTION

Maria: Conceptualization, Methodology, Formal analysis, Bayesian modeling, Software (MATLAB), Data curation, Visualization, Writing—original draft. Rian: Methodology review, Statistical validation, Interpretation of results, Writing—review & editing. Etriana: Data acquisition, Investigation, Educational context analysis, Visualization support, Writing—review.

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COMPETING INTEREST

The authors declare that they have no competing interests.

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