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Simulating the Dynamics of Early Marriage and Marital Stability Using SERH Mathematical Models

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ABSTRACT

This study aims to develop a mathematical model of SERH (Susceptible, Engaged, Risk, and Stabilization) to analyze and predict the incidence of early marriage in South Sulawesi Province. The research employs method a combination of theoretical and applied approaches, utilizing differential equations to model the dynamics of early marriage spread. Data were collected through questionnaires distributed to 400 couples selected using the Slovin technique, representing a population of 57,789 couples. The SERH model parameters, including the rate of couple interaction (α), transition from engagement to risk (β), and recovery from risk to stability (γ), were derived from the collected data. Simulations were conducted using Maple software to visualize the spread of early marriage under different scenarios. The analysis revealed two equilibrium points: a marriage-free equilibrium and a stable endemic equilibrium. The basic reproduction number (R_0) was calculated to be 3.97, indicating that one couple can influence 3 to 4 others in their social environment. However, with effective interventions such as education and counseling, the R_0 value can be reduced to 0.45, significantly lowering the spread of early marriage. This study provides valuable insights for policymakers to design targeted prevention programs and highlights the importance of early intervention in reducing the prevalence of early marriage.

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A. INTRODUCTION

Early marriage, defined as a practice in which one or both partners marry before reaching the age of 18, is a global social and health problem affecting millions of girls worldwide, particularly in developing countries (Singhal, 2020; UNICEF, 2020a; UNICEF, 2020b). According to UNICEF (2020a); UNICEF (2020b), the prevalence of early marriage in rural areas has decreased by only 5.76% over the past decade, while the decline in urban areas is even slower, at just 1%. This suggests that the rate of decline in early marriage prevalence is slowing down. In 2015, West Java recorded the highest absolute number of child marriages before the age of 18, with 273,000 cases reported in the province. Globally, UNICEF (2020a) reports that 1 in 9 girls are married before the age of 18, compared to only 1 in 100 boys. Factors contributing to early marriage include low household income, rural residence, and limited educational opportunities.

The impact of early marriage is profound and far-reaching, encompassing serious health complications and increased mortality rates. Babies born to mothers who marry at an early age are twice as likely to die in infancy (UNICEF, 2020a; UNICEF, 2020b). Additionally, early marriage significantly increases the risk of domestic violence (UNICEF, 2020a; UNICEF, 2020b). These consequences highlight the urgent need for national and international communities to address this issue. This study focuses on the phenomenon of early marriage in South Sulawesi Province, aiming to deepen the understanding of its root causes and impacts. The study seeks to analyze and simulate potential social cases of early marriage in South Sulawesi, with a particular emphasis on the factors influencing its prevalence in the region.

Previous studies on early marriage have primarily focused on its social and health impacts, often employing qualitative methods or descriptive analyses. For instance, research by UNICEF (2020a); UNICEF (2020b) highlights the global consequences of early marriage, such as increased infant mortality rates and domestic violence, while Singhal (2020) emphasizes the long-term health risks for young mothers and their children. Similarly, studies conducted in Indonesia, such as those by Agustina & Yusran (2024); Anjarsari & Sari (2020); Dinda et al. (2024) focus on the role of government and stakeholders in addressing early marriage and its impact on stunting growth rates. However, these studies lack a quantitative approach to predict the spread of early marriage or evaluate the effectiveness of interventions.

This study fills these gaps by developing a mathematical model (SERH) to simulate the spread of early marriage and assess the impact of interventions such as education and counseling. This study aims to develop a mathematical model of SERH (Susceptible, Engaged, Risk, and Stabilization) to analyze and predict the incidence of early marriage in South Sulawesi Province. By focusing on South Sulawesi, this research provides localized insights that can inform targeted policy interventions. Furthermore, the use of differential equations and simulation software (Maple) enables a more precise analysis of the dynamics of early marriage, making a novel contribution to the field.

B. RESEARCH METHOD

This study employs a mathematical modeling approach to analyze and predict the incidence of early marriage in South Sulawesi Province. The research was conducted in South Sulawesi, with a study population of approximately 57,789 couples. The first step in this study involved developing a mathematical model, specifically the SERH (Susceptible, Engaged, Risk, and Stabilization) model, to simulate the distribution of early marriage cases based on relevant data and variables in the region. The model was designed to identify key variables that contribute to early marriage, such as education level, location of residence, and socio-economic conditions (Bezie & Addisu, 2019).

Data were collected through questionnaires distributed to 400 couples selected using the Slovin technique, ensuring a representative sample of the population. The Slovin technique was chosen to minimize sampling error while maintaining a manageable sample size (Singhal, 2020). The questionnaire included questions related to socio-economic status, educational level, and cultural factors that influence early marriage. The collected data were then used to determine the parameters of the SERH model, including the rate of couple interaction (α), transition from engagement to risk (β), and recovery from risk to stability (γ). These parameters were derived from the responses and validated through statistical analysis.

After the model was developed, the analysis stage involved reviewing the equilibrium points of the model, evaluating stability based on eigenvalues, and calculating the basic reproduction number (R_0). The R_0 value is a critical metric used to measure the potential for the spread of early marriage in the population, where higher values indicate a faster and wider spread (Abdy et al., 2021; Annas et al., 2020; Side & Noorani, 2015; Side et al., 2018). The stability of the model was assessed using the Jacobian matrix, and the eigenvalues were calculated to determine whether the equilibrium points were stable or unstable.

The final stage of this study involved simulating the SERH model using Maple software to describe the dynamics of early marriage spread in South Sulawesi Province. The simulation aimed to visualize the spread scenario based on the specified parameters, including the impact of interventions such as education and counseling. The simulation results were used to evaluate the effectiveness of these interventions in reducing the prevalence of early marriage. The use of Maple software allowed for precise numerical simulations and graphical representations of the model's predictions (Abdy et al., 2021; Maryam et al., 2021; Rangkuti et al., 2014; Side & Noorani, 2012).

This study is expected to make a significant contribution to understanding the factors causing early marriage and provide useful information for policymakers and the community to address this issue. By combining theoretical modeling with empirical data, this research offers a comprehensive approach to analyzing and mitigating the spread of early marriage in South Sulawesi. All stages can be depicted in the following diagram (see Figure 1).

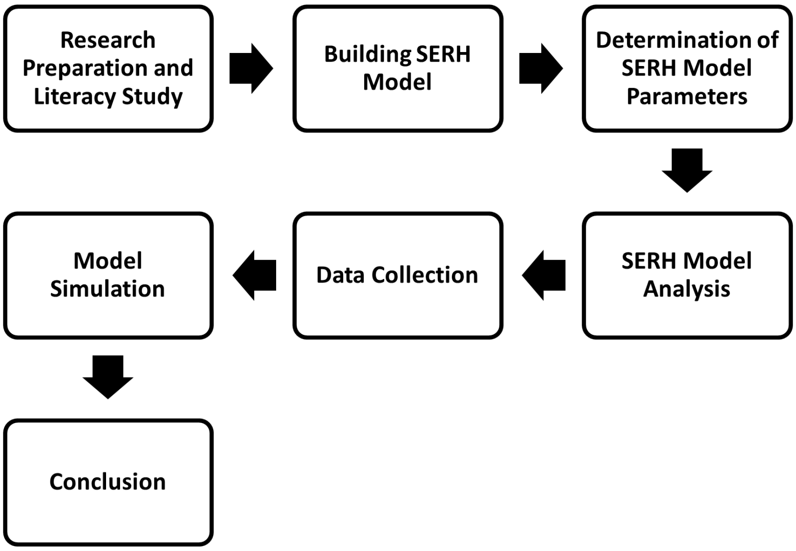

Figure 1. SERH Model Scheme for the Social Case of Early Marriage

C. RESULT AND DISCUSSION

1. Formation of SERH Model Social Case of Early Marriage

In this study, the population is categorized into four classes based on their respective epidemiological statuses within the SERH model: Susceptible Class (S), Engaged Class (E), Risk Class (R), and Stabilization Class (H). The Susceptible Class (S) comprises couples who are at risk of early marriage, totaling 3,356 couples. The Engaged Class (E) represents couples who have been involved in early marriage but are still in the early stages of their relationship, where adjustment and adaptation are ongoing, with a total of 982 couples. The Risk Class (R) includes couples who are in a vulnerable condition, facing conflict, instability, or potential separation due to factors such as age, emotional readiness, economic challenges, or social pressure, with a total of 399 couples. Finally, the Stabilization Class (H) comprises couples who have successfully navigated the critical phase and achieved stability in their relationship, characterized by understanding, support, and effective adaptation to the dynamics of early marriage, with a total of 53,052 couples. The total number of couples in this study is 57,789, spread throughout South Sulawesi.

The determination of the parameter rates (α, β, γ, and μ) is based on data collected through questionnaires distributed to 400 couples selected randomly using the Slovin technique. These parameters are essential for simulating the dynamics of early marriage spread using the SERH model. The schematic of the SERH model is presented in Figure 2, while the model definition and parameter values are detailed in Table 1 and 2, respectively.

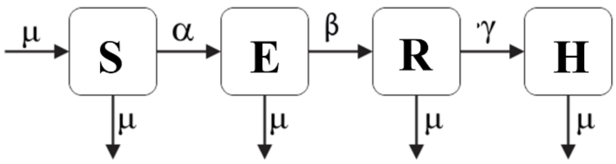

Figure 2. SERH Model Scheme for the Social Case of Early Marriage

Table 1. SERH Model Parameters Social Cases of Early Marriage

Variable/ Parameter	Definition
$S(t)$	Couples who are in an age range or social environment where early marriage is common, but have not yet engaged in a marital relationship.
$E(t)$	Couples who have been involved in early marriage, but their relationship is still in the early stages, where adjustments and adaptations are taking place.

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Variable/ Parameter	Definition
$R(t)$	Couples who are in a vulnerable condition towards conflict, instability, or potential separation, either due to age, emotional readiness, economic factors, or social pressures and ultimately end in divorce.
H(t)	Couples who successfully pass through the critical phase and achieve stability in their relationship are characterized by understanding, support, and effective adaptation to the dynamics of early marriage.
N	Total population
α	Factors that encourage couples to enter early marriage, including social pressure, cultural norms, and economic factors, are the rate at which couples meet.
β	The Rate of Couples Meeting Factors that influence couples to move from the "Engaged" to "Risk" stage. These can include emotional immaturity, lack of family support, or economic challenges faced by young couples.
γ	Couples' pace that meets the Factors that can help couples move out of the risk phase to stability. This includes local wisdom-based interventions such as community support, customary roles, and guidance from community or family leaders.
μ	Birth and Death Rates

$N = S + E + L + R$, where N is the total number of pairs.

Table 2. Results of Data Analysis Related to SERH Model Parameters for Social Cases of Early Marriage

Parameter	Data	Source
α	0.5	
β	0.225	Questionnaire/Haunted
γ	0.1	
μ	0.17	BPS 2023

Previous studies on early marriage have primarily focused on its social and health impacts, often using qualitative methods or descriptive analyses. For example, research by UNICEF (2020a); UNICEF (2020b) highlights the global consequences of early marriage, such as increased infant mortality rates and domestic violence, while Singhal (2020) emphasizes the long-term health risks for young mothers and their children. Similarly, studies conducted in Indonesia, such as those focus on the role of government and stakeholders in addressing early marriage and its impact on stunting growth rates (Agustina & Yusran, 2024; Anjarsari & Sari, 2020; Bezie & Addisu, 2019; Dinda et al., 2024). However, these studies lack a quantitative approach to predict the spread of early marriage or evaluate the effectiveness of interventions.

2. Equilibrium Point Analysis SERH Model

Based on Figure 2, the SERH mathematical model is obtained. This can be seen in Equations (1) to (4):

$$\frac{dS}{dt} = \mu - \alpha SR - \mu S \tag{1}$$
$$\frac{dE}{dt} = \alpha S - \beta E - \mu E \tag{2}$$
$$\frac{dR}{dt} = \beta E - \gamma R - \mu R \tag{3}$$
$$\frac{dH}{dt} = \gamma R - \mu H \tag{4}$$

with $N = S + E + R + H$.

3. Determination Point Equilibrium

Equations (1) to (4) have equilibrium points, which are solutions to $\frac{dS}{dt}, \frac{dE}{dt}, \frac{dR}{dt}, \frac{dH}{dt} = (0, 0, 0, 0)$. If the right-hand side of the equation is equated to zero, then equations (5) to (8) are obtained, namely:

$$\frac{dS}{dt} = \mu - \alpha SR - \mu S = 0 \tag{5}$$
$$\frac{dE}{dt} = \alpha S - \beta E - \mu E = 0 \tag{6}$$
$$\frac{dR}{dt} = \beta E - \gamma R - \mu R = 0 \tag{7}$$
$$\frac{dH}{dt} = \gamma R - \mu H = 0 \tag{8}$$

Equilibrium point in the SERH model. In this, two equilibrium point conditions are formed, namely the free equilibrium point and the equilibrium point.

4. Free Equilibrium Point

Free can be assumed that there is no population. The free equilibrium point occurs if $R = 0$. In this situation, Equations (5) to (8) are used to represent the equilibrium as follows.

Trait 1

Free equilibrium points of the SERH model are given as:

$$(S, E, R, H) = \left(\frac{\mu}{\mu},\ 0, 0, 0\right)$$

Therefore, based on the free equilibrium points for the SERH model, after substituting the parameter values in Table 1, it is $(S, E, R, H) = (1, 0, 0, 0)$.

Proof:
By making $R = 0$ Equation (7), we obtain Equations (9) to (12):

$$\begin{aligned} &R = 0 \\ &\mu - \alpha SR - \mu S = 0 \end{aligned} \tag{9}$$

$$S = \frac{\mu}{\mu} = 1 \tag{10}$$

$$\begin{aligned} &\alpha SR - \beta E - \mu E = 0 \\ &E = \tfrac{\alpha SR}{\beta+\mu} = \tfrac{0}{\mu} = 0 \end{aligned} \tag{11}$$

$$\begin{aligned} &\gamma R - \mu H = 0 \\ &R = \tfrac{\gamma R}{\mu} = \tfrac{0}{\mu} = 0 \end{aligned} \tag{12}$$

So, we get an equilibrium point, which is symbolized by

$$(S, E, R, H) = \left(\frac{\mu}{\mu},\ 0, 0, 0\right) \tag{13}$$

5. Non-Free Equilibrium Point

The non-free equilibrium point is determined by setting the left-hand side of Equations (5) to (8) equal to zero and solving for the equilibrium values of the variables S*, E*, R*, H*.

Trait 2

Endemic equilibrium point of SERH model given as:

$$(S*, E*, R*, H*) = \begin{pmatrix} \frac{(\gamma+\mu)(\beta+\mu)\alpha}{\beta}, \\ \frac{\mu(\beta\alpha-(\gamma+\mu)(\beta+\mu))}{\alpha(\gamma+\mu)(\beta+\mu)}\left(\gamma+\mu\right)\beta, \\ \frac{\mu(\beta\alpha-(\gamma+\mu)(\beta+\mu))}{\alpha(\gamma+\mu)(\beta+\mu)} \\ \gamma\mu\frac{\mu(\beta\alpha-(\gamma+\mu)(\beta+\mu))}{\alpha(\gamma+\mu)(\beta+\mu)} \end{pmatrix}$$

Proof:

Step 1, from Equation (7):

$$\begin{aligned} &\beta E - \gamma R - \mu R = 0 \\ &\beta E \quad = R\left(\gamma+\mu\right) \\ &E \quad = R\left(\gamma+\mu\right)\beta \end{aligned} \tag{14}$$

Step 2, solving Equation (6) for S. From Equation (6):

$\alpha SR - \beta E - \mu E = 0$

Substitute E from Step 1:

$$\begin{aligned} \alpha SR - \beta\left(\frac{R(\gamma+\mu)}{\beta}\right) - \mu\left(\frac{R(\gamma+\mu)}{\beta}\right) = 0 \\ \alpha SR - R\left(\gamma+\mu\right) - \frac{\mu R(\gamma+\mu)}{\beta} = 0 \end{aligned} \tag{15}$$

Factor R:

$$R\left(\alpha S - (\gamma+\mu) - \frac{\mu\left(\gamma+\mu\right)}{\beta}\right) = 0 \tag{16}$$

Since $R \neq 0$ (non-disease-free condition), then:

$$\begin{aligned} \alpha S &= (\gamma+\mu) + \frac{\mu(\gamma+\mu)}{\beta} \\ \alpha S &= (\gamma+\mu)\left(1+\frac{\mu}{\beta}\right) \\ \alpha S &= (\gamma+\mu)\left(\beta+\frac{\mu}{\beta}\right) \\ S &= \frac{(\gamma+\mu)(\beta+\mu)}{\alpha\beta} \end{aligned} \tag{17}$$

Step 3, solving Equation (5) for R. From Equation (5):

$\mu - \alpha SR - \mu S = 0$

Substitute S from (16):

$$\mu - \alpha(\frac{(\gamma+\mu)\left(\beta+\mu\right)}{\alpha\beta})R - \mu(\frac{(\gamma+\mu)\left(\beta+\mu\right)}{\alpha\beta}) = 0 \tag{18}$$

Simplify:

$$\begin{aligned} &\mu - \frac{(\gamma+\mu)(\beta+\mu)}{\beta}R - \frac{\mu(\gamma+\mu)(\beta+\mu)}{\alpha\beta} = 0 \\ &\mu \quad = \quad \frac{(\gamma+\mu)(\beta+\mu)}{\beta}R + \frac{\mu(\gamma+\mu)(\beta+\mu)}{\alpha\beta} \\ &\qquad \mu - \frac{\mu(\gamma+\mu)(\beta+\mu)}{\alpha\beta} = \frac{(\gamma+\mu)(\beta+\mu)}{\beta}R \\ &R \quad = \quad \mu - \frac{\frac{\mu(\gamma+\mu)(\beta+\mu)}{\alpha\beta}}{\frac{(\gamma+\mu)(\beta+\mu)}{\beta}} \\ &R \quad = \quad \mu\beta\alpha - \frac{\mu(\gamma+\mu)(\beta+\mu)}{\alpha(\gamma+\mu)(\beta+\mu)} \\ &R \quad = \quad \frac{\mu(\beta\alpha-(\gamma+\mu)(\beta+\mu))}{\alpha(\gamma+\mu)(\beta+\mu)} \end{aligned} \tag{19}$$

Step 4, solving Equation (8) for H. From Equation (8):

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$$\begin{aligned} \gamma R - \mu H = 0 \\ H = \gamma R \mu \end{aligned} \tag{20}$$

Substitute R from Step 3:

$$\begin{aligned} H &= \frac{\gamma}{\mu} \cdot \frac{\mu(\beta\alpha - (\gamma+\mu)(\beta+\mu))}{\alpha(\gamma+\mu)(\beta+\mu)} \\ H &= \frac{\gamma(\beta\alpha - (\gamma+\mu)(\beta+\mu))}{\alpha(\gamma+\mu)(\beta+\mu)} \end{aligned} \tag{21}$$

Equilibrium Point (S^*, E^*, R^*, H^*). After solving the system of equations, the equilibrium point is:

$$(S*, E*, R*, H*) = \begin{pmatrix} \frac{(\gamma+\mu)(\beta+\mu)\alpha}{\beta}, \\ \frac{\mu(\beta\alpha-(\gamma+\mu)(\beta+\mu))}{\alpha(\gamma+\mu)(\beta+\mu)} (\gamma+\mu)\beta, \\ \frac{\mu(\beta\alpha-(\gamma+\mu)(\beta+\mu))}{\alpha(\gamma+\mu)(\beta+\mu)}, \\ \gamma\mu \frac{\mu(\beta\alpha-(\gamma+\mu)(\beta+\mu))}{\alpha(\gamma+\mu)(\beta+\mu)} \end{pmatrix} \tag{22}$$

Then, the equilibrium point for the social case is after the parameter values in Table 2. Substituted is $(S*, E*, R*, H*) = (0.948, 0.0186, 0.0186, 0.00186)$.

6. Stability of SERH Model

Based on Equations (1)-(4), the Jacobian matrix (J) can be formed as follows:

$$J = \begin{bmatrix} -\alpha L - \mu & 0 & -\alpha s & 0 \\ \alpha R & -\beta - \mu & \alpha s & 0 \\ 0 & \beta & -\gamma - \mu & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix} \tag{23}$$

Theorem

The equilibrium point in the SERH model is considered stable if the basic reproduction number (R_0) is less than or equal to 1 ($R_0 \leq 1$), and is considered unstable if R_0 is greater than 1 ($R_0 > 1$).

Proof

The equilibrium points are inserted into the J matrix in Equation (23),

$$J = \begin{bmatrix} -\alpha L - \mu & 0 & -\alpha s & 0 \\ \alpha L & -\beta - \mu & \alpha s & 0 \\ 0 & \beta & -\beta - \mu & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}$$

Then, the eigenvalues of the matrix in Equation (23) are determined, with the following description:

$$\begin{aligned} &|\lambda I - J| = 0 \\ &|\lambda I - J| = \left| \left(\lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -\alpha R - \mu & 0 & -\alpha s & 0 \\ \alpha R & -\beta - \mu & \alpha s & 0 \\ 0 & \beta & -\gamma - \mu & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix} \right) \right| = 0 \\ &|\lambda I - J| = \left| \begin{bmatrix} -\alpha L - \mu - \lambda & 0 & \alpha s & 0 \\ \alpha R & -\beta - \mu - \lambda & \alpha s & 0 \\ 0 & \beta & -\gamma - \mu - \lambda & 0 \\ 0 & 0 & \gamma & \mu - \lambda \end{bmatrix} \right| \end{aligned} \tag{24}$$

Next, the determinant will be calculated, resulting in:

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$$
\begin{aligned}
&\alpha\beta\gamma L\lambda+\alpha\beta\gamma L\mu+\alpha\beta L\lambda^2+2\alpha\beta L\lambda\mu+\alpha\beta L\mu^2-\alpha\beta\lambda^2 s-2\alpha\beta\lambda\mu s-\alpha\beta\mu^2 s\\
&+\alpha\gamma L\lambda^2+2\alpha\gamma L\lambda\mu+\alpha\gamma L\mu^2+\alpha L\lambda^3+3\alpha L\lambda^2\mu+3\alpha L\lambda\mu^2+\alpha L\mu^3\\
&+\beta\gamma\lambda^2+2\beta\gamma\lambda\mu+\beta\gamma\mu^2+\beta\lambda^3+3\beta\lambda^2\mu+3\beta\lambda\mu^2+\beta\mu^3\\
&+\gamma\lambda^3+3\gamma\lambda^2\mu+3\gamma\lambda\mu^2+\gamma\mu^3+\lambda^4+4\lambda^3\mu+6\lambda^2\mu^2+4\lambda\mu^3+\mu^4
\end{aligned} \tag{25}
$$

According to Descartes' sign rule, Equation (25) will have only negative roots if all terms have positive signs. Thus, it can be concluded that the equilibrium point of the given model is stable when $R_0 \leq 1$ and becomes unstable when $R_0 \geq 1$.

7. Eigenvalues

Referring to Equation (25), the eigenvalues (λ) Derived from the previous equilibrium point, these values are real and negative. Based on stability properties, this indicates that the equilibrium point is asymptotically stable. Consequently, the stability of the SERH Model for the social case of early marriage, represented by $(S,\ E,\ R,\ H) = (-0.04067818969,\ -0.3831813677,\ -0.1339984110,\ -0.1339984110)$, confirms that both equilibrium points exhibit asymptotic stability.

8. Basic Reproduction Number

The basic reproduction number R_0 is derived using the next-generation matrix method. This matrix is constructed by analyzing the positive and negative components of the transmission rate in the SERH model. The formula for calculating R_0 is provided in Equation (24).

$$R = F\prime \cdot (V\prime)^{-1} \tag{26}$$

Based on the Equation:

$$\frac{dE}{dt} = \alpha SR - \beta E - \mu E = 0 \tag{27}$$

$$\frac{dR}{dt} = \beta E - \gamma R - \mu R = 0 \tag{28}$$

So that it is obtained:

$$F = \begin{bmatrix} \alpha SR \\ 0 \end{bmatrix}, F\prime = \begin{bmatrix} 0 & \alpha S \\ 0 & 0 \end{bmatrix} \tag{29}$$

$$V = \begin{bmatrix} \beta E + \mu E \\ \gamma I + \mu I + \beta E \end{bmatrix}, V_h\prime = \begin{bmatrix} \beta+\mu & 0 \\ \beta & \gamma+\mu \end{bmatrix} \tag{30}$$

Then, the inverse of the matrix in Equation (30) is obtained as follows:

$$(V\prime)^{-1} = \begin{bmatrix} \frac{1}{\beta+\mu} & 0 \\ \frac{\beta}{(\beta+\mu)(\gamma+\mu)} & \frac{1}{\gamma+\mu} \end{bmatrix} \tag{31}$$

Next, the eigenvalues of the R matrix will be calculated according to Equation (31).

$$
\begin{aligned}
R &= \begin{bmatrix} 0 & \alpha S \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\beta+\mu} & 0 \\ \frac{\beta}{(\beta+\mu)(\gamma+\mu)} & \frac{1}{\gamma+\mu} \end{bmatrix}\\
R &= \begin{bmatrix} \alpha S\beta & \frac{\alpha S}{\gamma+\mu} \\ 0 & 0 \end{bmatrix}
\end{aligned} \tag{32}
$$

After obtaining the matrix R in Equation (32), the eigenvalues are determined using the characteristic equation $Det\,(\lambda I - R) = 0$, where I represents the identity matrix. The basic reproduction number (R_0) is then identified as the largest eigenvalue (λ).

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$$|\lambda I - R| = \left| \left(\lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{bmatrix} \frac{\alpha S \beta}{(\beta+\mu)(\gamma+\mu)} & \frac{\alpha S}{\gamma+\mu} \\ 0 & 0 \end{bmatrix} \right) \right| = 0 \tag{33}$$

So by using the matrix addition operation above, two lambda values are obtained, which are eigenvalues based on Equation (33), namely:

$$\lambda_{\,1,2} = \left[\frac{\alpha S \beta}{(\beta+\mu)(\gamma+\mu)},\ 0 \right]$$

So the lambda value above is the largest eigenvalue, namely:

$$\lambda = \left[\frac{\alpha S \beta}{(\beta+\mu)(\gamma+\mu)} \right]$$

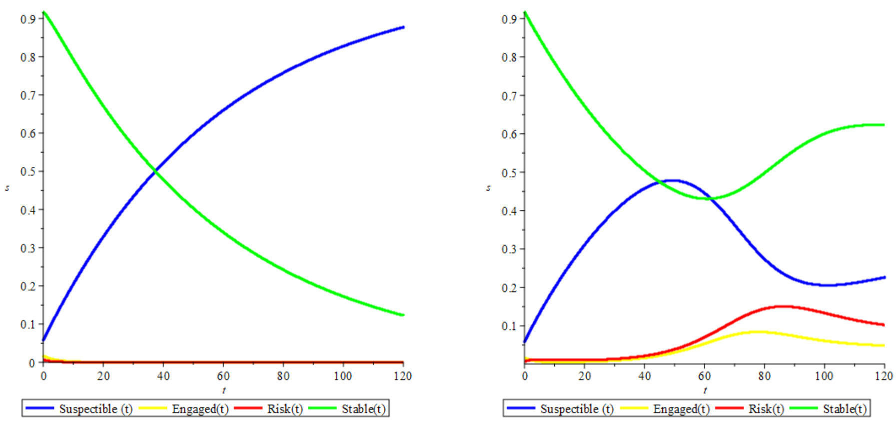
with the largest lambda value compared to λ_2 the assumption of parameter rates and the presence of a population in the model. So based on the largest eigenvalue substituted for the free equilibrium point value, the basic reproduction number is obtained:

$$R_0 = \frac{\alpha \mu \beta}{(\beta+\mu)(\gamma+\mu)\,\mu} \tag{34}$$

Based on the equation above, the basic reproduction number for cases without a solution will produce $R_{0n} = 3.973299428$ which means in social cases where one couple can influence 3-4 couples around them, but on the other hand, if the case is given a solution, then it will produce $R_{0s} = 0.4571052439$ which means that there is no psychological spread, where each couple does not influence other couples.

9. Simulation Results of the SERH Model for Social Cases of Early Marriage

In this case, a simulation will be conducted based on the successfully collected data, as illustrated in the following graph. The simulation of the SERH mathematical model is illustrated in Figures 3a & 3b.



(a) (b)

Figure 3. Plot of SERH Model without (a) and with (b) Solution to the Social Case of Early Marriage

The simulation results of the SERH model provide valuable insights into the dynamics of early marriage spread in South Sulawesi Province. The model was simulated using Maple software, and the results are presented in Figures 2. The simulation was conducted under two scenarios: (a) without intervention and (b) with intervention, such as education and counseling programs. The results highlight the effectiveness of interventions in reducing the prevalence of early marriage and stabilizing relationships among young couples.

In Figure 3a which represents the simulation without intervention, the number of couples in the Susceptible class (S) increased from an initial observation of 0.06 to 0.5 over 60 months, followed by a decrease to 0.23 in the next 40 months. This fluctuation indicates that without intervention, the number of couples at risk of early marriage remains high and unstable. The

Engaged class (E), representing couples in the early stages of marriage, showed a significant 100% increase within 65 months, followed by a gradual decline. This suggests that early marriages are prone to instability without proper support. The Risk class (R), which includes couples facing conflict or potential separation, increased by nearly 150% within 90 months, reaching a peak of 0.15 before declining. This highlights the vulnerability of young couples to social and economic pressures. Finally, the Stabilization class (H), representing couples who have achieved stability in their relationships, decreased by 50% within 60 months, indicating that without intervention, the likelihood of achieving long-term stability is low.

In contrast, Figure 3b shows the simulation results with intervention. The number of couples in the Susceptible class (S) increased from 0.06 to 0.89 over 120 months, indicating that interventions such as education and counseling can significantly reduce the number of couples at risk of early marriage. The Engaged class (E) and Risk class (R) remained below 5% of the population throughout the simulation, demonstrating that interventions can effectively stabilize relationships and reduce the likelihood of conflict or separation. The Stabilization class (H) showed a 70% decrease over 120 months, indicating that interventions can help couples achieve long-term stability in their relationships.

The basic reproduction number (R_0) calculated from the SERH model, these findings are further supported. Without intervention, the (R_{0n}) value was 3.97, indicating that one couple can influence 3-4 others in their social environment. However, with intervention, the (R_{0s}) value decreased to 0.45, suggesting that the spread of early marriage can be effectively controlled through targeted programs (Abdy et al., 2021; Annas et al., 2020; Nurhaeda et al., 2021; Rangkuti et al., 2014; Side & Noorani, 2012). These results align with previous studies that emphasize the importance of early intervention in reducing the prevalence of early marriage (Bezie & Addisu, 2019).

Overall, the simulation results demonstrate the potential of the SERH model to predict the spread of early marriage and evaluate the effectiveness of interventions. By providing a quantitative framework for analyzing the dynamics of early marriage, this study offers policymakers and stakeholders valuable insights for designing targeted prevention programs. months.

10. Discussion

The simulation results of the SERH model provide critical insights into the dynamics of early marriage in South Sulawesi Province. While previous studies have primarily focused on the social and health impacts of early marriage using qualitative methods or descriptive analyses (Annas et al., 2020; Bezie & Addisu, 2019; Singhal, 2020), this study fills a significant gap by employing a quantitative approach through mathematical modeling. Unlike earlier research, which often lacked predictive capabilities or the ability to evaluate the effectiveness of interventions, the SERH model offers a robust framework for simulating the spread of early marriage and assessing the impact of various prevention strategies. This approach enables a deeper understanding of the underlying mechanisms driving early marriage and provides policymakers with actionable insights.

The findings reveal that early marriage is a complex social issue influenced by various factors, including socio-economic conditions, cultural norms, and lack of education. Without intervention, the number of couples at risk of early marriage remains high, with significant fluctuations observed in the Susceptible (S), Engaged (E), and Risk (R) classes. This aligns with previous studies that highlight the persistent prevalence of early marriage in developing regions, particularly in rural areas with limited access to education and economic opportunities (Bezie & Addisu, 2019).

The simulation without intervention (Figure 3a) shows that the number of vulnerable couples decreases slightly in the short term but increases significantly over time. For instance, the Susceptible class (S) experienced a decline from 24% to 19% within 12 months, but the Engaged class (E) and Risk class (R) showed a sharp increase, indicating that social pressures and economic challenges continue to drive early marriage. This finding is consistent with research by (Singhal, 2020), which emphasizes that early marriage often stems from poverty, lack of education, and cultural practices that prioritize early unions. The increase in the Risk class (R) also underscores the vulnerability of young couples to conflict, instability, and potential separation, which are common consequences of early marriage (UNICEF, 2020a; UNICEF, 2020b).

In contrast, the simulation with intervention (Figure 3b) demonstrates the effectiveness of targeted programs such as education, counseling, and community support. The number of couples in the Susceptible class (S) increased from 23% to 29% within 12 months, indicating that interventions can reduce the number of couples at risk of early marriage. The Engaged class (E) and Risk class (R) remained below 5% of the population, suggesting that interventions can stabilize relationships and reduce the likelihood of conflict or separation. These results are supported by studies that highlight the positive impact of education and counseling in delaying marriage and improving marital stability (Bezie & Addisu, 2019).

The simulation also highlights the importance of timely intervention. A 3-month delay in implementing solutions led to a significant increase in the number of vulnerable couples, with the Susceptible class (S) reaching 100% within 6 months before stabilizing. This finding underscores the urgency of addressing early marriage through proactive measures, as delays can exacerbate the problem and make it more challenging to control (Saman et al., 2022). The basic reproduction number (R_0) further supports this conclusion. Without intervention, the (R_{0n}) value was 3.97, indicating that one couple can influence 3-4 others in their social environment. However, with intervention, the (R_{0s}) value decreased to 0.45, demonstrating that targeted programs can effectively reduce the spread of early marriage (Abdy et al., 2021; Annas et al., 2020).

The findings of this study have important implications for policymakers and stakeholders. The SERH model provides a quantitative framework for understanding the dynamics of early marriage and evaluating the effectiveness of interventions. By identifying key factors that contribute to early marriage, such as low education levels and economic challenges, this study offers valuable insights for designing targeted prevention programs. For example, increasing access to education, providing economic opportunities for young women, and promoting community-based counseling programs can significantly reduce the prevalence of early marriage and its negative impacts (Bezie & Addisu, 2019).

Research by Bezie & Addisu (2019), UNICEF (2020a), UNICEF (2020b), and Singhal (2020) has identified factors influencing early marriage, such as low levels of education, social, cultural, and economic pressures. These studies indicate that early marriage leads to high infant mortality rates, the risk of domestic violence, and a decline in the quality of health and education. However, the approaches used are largely qualitative and descriptive. This study, however, develops a quantitative mathematical model based on the SERH (Susceptible, Engaged, Risk, Stabilization) model to predict the spread and effectiveness of interventions aimed at preventing early marriage in South Sulawesi. Simulation results indicate that without intervention, the basic reproduction number (R_0) reaches 3.97, suggesting that one couple can potentially influence 3–4 other couples to marry early. However, with interventions such as education and counseling, the R_0 value can be reduced to 0.45. These results indicate that this social classification model is superior in mapping the dynamics of the spread of early marriage, as well as producing a population distribution graph for each class (S, E, R, H), which shows the model's sensitivity in the process of preventing and stabilizing relationships.

D. CONCLUSION AND SUGGESTION

The conclusion of this study demonstrates that the SERH mathematical model effectively identified patterns and risks associated with early marriage in South Sulawesi. The simulation revealed critical points in young couples' relationships, especially in the early stages of marriage, which are prone to conflict. Interventions such as education, counseling, and environmental support have proven effective in reducing the negative impacts of early marriage, as indicated by the decrease in the R_0 value. This study highlights the importance of a comprehensive approach to preventing early marriage and its long-term consequences, and offers guidance for the development of more effective policies.

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DECLARATIONS

AUTHOR CONTIBUTION

All authors contributed to this manuscript, from exploring ideas to writing this article.

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COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

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